INDIAN KNOWLEDGE SYSTEM AND THE LEGACY OF SRINIVASA RAMANUJAN

Dr. Utpal Pore
Department of statistics, Faculty of Science, St. Xavier's University, Kolkata
utpal.mathju@gmail.com, utpal.pore@sxuk.edu.in

Abstract
The contributions of Indian mathematicians began with the foundational work of the post-Vedic era. The scholarly Rishis of this period systematized the study of mathematics, geometry, and trigonometry. This rich tradition found its most brilliant expression in the modern era through the unparalleled genius of S. Ramanujan, whose profound insights into pure mathematics earn him a place among the most revered figures in the field, such as Gauss, Euler, and Riemann. In this review article comprehensively examines Ramanujan's seminal work in Partition Theory, the Hardy-Ramanujan Circle Method, and the enigmatic Mock Theta Functions. Furthermore, we explore the enduring impact and applications of these contributions within modern mathematical sciences, highlighting their continued relevance and inspirational value for contemporary research.
Keywords: *Indian Knowledge System, Vedic Mathematics, Circle method, Mock theta function, Modular Forms*

1. Introduction

Some concepts are so fundamental that they get absorbed in the general thinking of all the peoples of the world and it is indeed hard to pinpoint a specific period for their birth. One such is the basic idea of introducing the place value system and the number zero, the credit for which goes to Ancient India. In the history of mathematics, Indian Mathematicians contributed very pioneer concepts such as the invention of zero, the invention of the decimal notation and creation of modern arithmetic; the invention of the sine and cosine functions leading to the creation of modern trigonometry; Diophantine equations, square roots, cube roots, and negative numbers and creation of algebra (Dutta, 2002).

This illustrious journey began in the Vedic era with the Śulba Sūtras (c. 800 BCE), where scholars like Baudhāyana established geometric principles for altar construction. The classical period (c. 500–1200 CE) witnessed the groundbreaking work of Āryabhaṭa in astronomy and trigonometry, and Brahmagupta's formalization of arithmetic operations with zero and negative numbers. Vedic Mathematics is an ancient Indian mathematical system of education which includes mathematical calculations such as addition, subtraction, multiplication, division, and solutions to geometric, algebraic, and trigonometric problems. It is defined by 16 Vedic Sūtras and 13 Sub-Sūtras with simple rules and concepts, later revived by Swami Bharati Krishna Tirtha (Tirtha, 1992). The Kerala School (14th–16th centuries), with luminaries like Mādhava, further advanced calculus through infinite series for π and trigonometric functions centuries before their European discovery (Balagangadharan, 1995). The mathematical legendary and highly influential genius S. Ramanujan $(1887 - 1920)$ has made tremendous impact on the development of the theory of partitions, a theory which was apparently first studied by L. Euler $(1707 - 1783)$ (Ramanujan, 1957). From ancient sutras to infinite series, these contributions form an indispensable chapter in the history of science.

2. Ramanujan's General Theta Function

At the advice of his teacher and friend, Seshu Aiyar, Ramanujan wrote a letter on January 13, 1913, to famous British mathematician G.H. Hardy, then Fellow of Trinity College, Cambridge. Enclosed also in this letter was a set of mathematical results including one hundred and twenty theorems. After receiving this material, Hardy discussed it with J.E. Littlewood about Ramanujan's mathematical talent. At the beginning Hardy was reluctant but impressed by Ramanujan's results on continued fractions. Finally, Hardy decided to bring Ramanujan to Cambridge to pursue some Joint research on mathematics.

Ramanujan's general theta function $f(a,b)$ is defined as:

$$f(a,b) = \sum_{n=-\infty}^{\infty} a^{n(n+1)/2} b^{n(n-1)/2}$$
$$= 1 + a + b + a^3b + ba^3 + a^6b^3 + a^3b^6 + \cdots$$

where $|ab| < 1$.

Ramanujan's pioneering investigations into theta functions provide foundational prototypes for several central themes in modern mathematics, including the proof of Fermat's Last Theorem, the development of the Langlands Program, and the formulation of the theory of Monstrous Moonshine with its applications to string theory (Berndt, 2006).

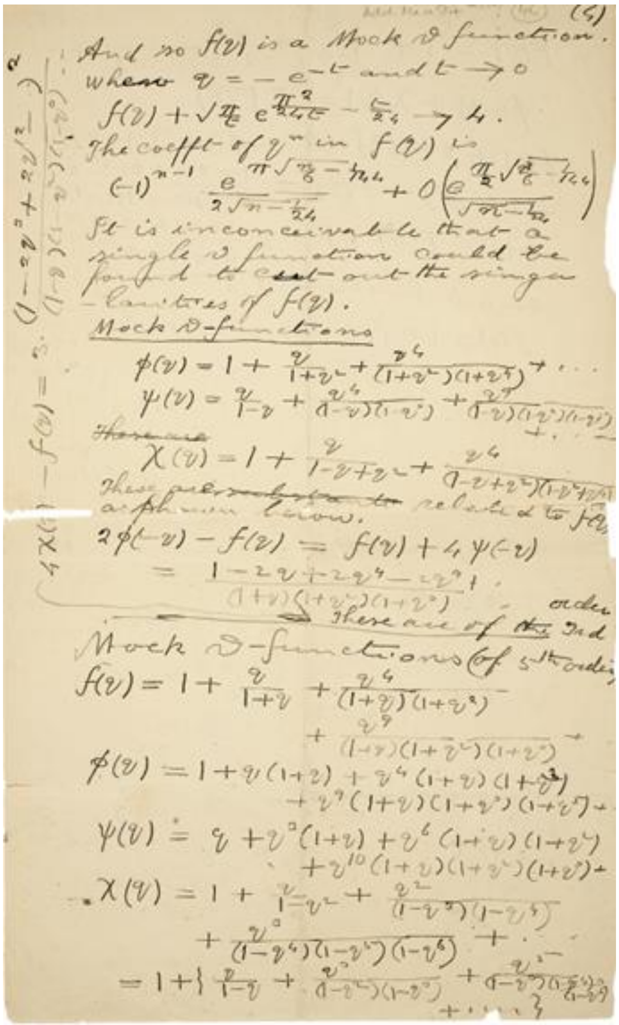
3. Srinivasa Ramanujan's Influence on Statistical Mechanics, String Theory and Black Holes

One of the most remarkable applications of the Ramanujan-Hardy asymptotic formula for $p(n)$ deals with the problems of statistical mechanics. Several authors including Auluck and Kotharl (Kothari, 1946), (Temperley, 1949) and Dutta (1956) discussed the significant role of partition functions in statistical mechanics. The theory of partitions of numbers has been found to be very useful for the study of the Bose-Einstein condensation of a perfect gas. The central problem is the determination of number of ways a given amount of energy can be shared out among different possible states of a thermodynamic assembly. This problem is essentially the same as that of finding the number of partitions of a number into integers under certain restrictions.

Ramanujan's mathematical ideas have deeply influenced modern physics, including string theory, and inspired many physicists. Nobel laureate S. Chandrasekhar greatly admired Ramanujan and helped preserve his legacy. Ramanujan's influence on physics continues to grow and remains important today.

In his last letter to Hardy, dated January 12, 1920, Ramanujan shared hints of his last theory. "I am extremely sorry for not writing you...I discovered very interesting functions recently which I call "Mock" theta functions.... they enter mathematics as beautifully as the ordinary theta functions. I am sending you with this letter some examples". Ramanujan's letter consists of 17 mock theta functions.

Ramanujan's work has had a profound influence on modern physics because modular forms are fundamental to string theory and two-dimensional conformal field theory (CFT). A closed string at a fixed moment can be viewed as a circle embedded in a higher-dimensional space. Ramanujan extensively studied modular and mock modular forms, many of which possess integer coefficients in their q-expansions and remain important objects in number theory. Interestingly, in several cases, the coefficients of these forms are closely connected to the characters of finite groups, revealing deep links between mathematics and physics.


And so $f(q)$ is a Mock ϑ function.
where $q = -e^{-t}$ and $t \to 0$
$f(q) + \sqrt{\frac{\pi}{t}}\, e^{\frac{\pi^2}{24t} - \frac{t}{24}} \to 4.$
The coeff of q^n in $f(q)$ is
$(-1)^{n-1} \frac{e^{\pi\sqrt{\frac{n}{6} - \frac{1}{144}}}}{2\sqrt{n - \frac{1}{24}}} + O\left(\frac{e^{\frac{\pi}{2}\sqrt{\frac{n}{6} - \frac{1}{144}}}}{\sqrt{n - \frac{1}{24}}}\right)$
It is inconceivable that a single ϑ function could be found to cut out the singularities of $f(q)$.
Mock ϑ-functions
$\phi(q) = 1 + \frac{q}{1+q^2} + \frac{q^4}{(1+q^2)(1+q^4)} + \cdots$
$\psi(q) = \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^9}{(1-q)(1-q^3)(1-q^5)} + \cdots$
$\chi(q) = 1 + \frac{q}{1-q+q^2} + \frac{q^4}{(1-q+q^2)(1-q^2+q^4)} + \cdots$
These are related to $f(q)$ as shown below.
$2\phi(-q) - f(q) = f(q) + 4\psi(-q)$
$= \frac{1 - 2q + 2q^4 - 2q^9 + \cdots}{(1+q)(1+q^2)(1+q^3)\cdots}$
These are of the 3rd order
Mock ϑ-functions (of 5th order)
$f(q) = 1 + \frac{q}{1+q} + \frac{q^4}{(1+q)(1+q^2)} + \frac{q^9}{(1+q)(1+q^2)(1+q^3)} + \cdots$
$\phi(q) = 1 + q(1+q) + q^4(1+q)(1+q^3) + q^9(1+q)(1+q^3)(1+q^5) + \cdots$
$\psi(q) = q + q^3(1+q) + q^6(1+q)(1+q^2) + q^{10}(1+q)(1+q^2)(1+q^3) + \cdots$
$\chi(q) = 1 + \frac{q}{1-q^2} + \frac{q^2}{(1-q^3)(1-q^4)} + \frac{q^3}{(1-q^4)(1-q^5)(1-q^6)} + \cdots$
$= 1 + \left\{\frac{q}{1-q} + \frac{q^2}{(1-q^2)(1-q^3)} + \frac{q^3}{(1-q^3)(1-q^4)(1-q^5)} + \cdots\right\}$

Page from Ramanujan's last letter to Hardy

At very high energies, the number of physical states grows rapidly, leading to a limiting temperature in systems such as string theory and black hole physics, a behaviour connected to the Hardy–Ramanujan $p(n)$ function describing integer partitions. Ramanujan's ideas, including mock modular forms, play a key role in modern physics by helping scientists understand and calculate the properties and entropy of black holes. Together, these developments show that Ramanujan's work continues to strongly influence present-day physics.

4. Conclusion

Ramanujan's achievements reflect the rich tradition of Indian mathematics. India has made significant contributions to mathematics through the development of the decimal system, the concept of zero, and advances in algebra and number theory. Ramanujan carried this legacy forward through his remarkable discoveries and his work continues to inspire mathematicians around the world.

References

1. Balagangadharan, K. (1995). Mathematical Analysis in Medieval Kerala, Science in the West and India, Some Historical Aspects, Edited by B.V. Subbarayappa and N. Mukunda. Himalaya Publishing House,.
2. Balagangadharan:, K. (1995). Mathematical Analysis in Medieval Kerala. Himalaya Publishing House.
3. Berndt, B. (2006). Number Theory in the Spirit of Ramanujan. American Mathematical Society, Providence.
4. Dutta, A. (2002). Mathematics in ancient india: An overview. Resonance, 270-283.
5. Kothari, F. C. (1946). Statistical mechanics and the partitions of numbers. Mathematical Proceedings of the Cambridge Philosophical Society, 42, , pp 272-277.
6. Ramanujan, S. (1919). Some properties of p(n), the number of partitions of n. Proc. Camb. Philos. Soc. 19, 207–210.
7. Ramanujan, S. (1957). Notebooks, 2 Volumes. Tata Institution of Fundamental.
8. Temperley, H. N. (1949). Statistical mechanics and the partition of numbers I. The transition of liquid helium . King’s College, University of Cambridge, pp 361-375.
9. Tirtha, S. B. (1992). Vedic mathematics (Vol. 10). Motilal Banarsidass Publ. .