REPRESENTATION FRAMEWORK FOR CARNATIC MUSIC MELODIES USING 22 SHRUTHIS

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Abstract

Mathematics is a wonderful science which has intricate weaving with almost all branches of arts and science. Music is a form of art present in living and non-living beings. It is a matter of great interest to understand the mathematics behind the theory of music. It is known that sound is produced when a string vibrates at a certain frequency. This frequency of vibration is inversely proportional to the length of the string. The seven notes of Carnatic music are Shadja (S), Rishabha (R), Gandhara (G), Madhyama (M), Panchama (P), Dhaivatha (D) and Nishada (N). The places or Swarasthanas of these notes in the frequency spectrum are defined according to the ratio of their frequency of vibration to that of the base note, generally S. These notes along with rhythm or taalas fill the hearts of music lovers with immense joy and take the music lovers to a different world. This paper aims at explaining the Carnatic music concept of twenty-two shrutis, using the fundamentals of sound such as wavelength and frequency.

Keywords – Carnatic music, 22 shruthis

Introduction

Nada is a musical sound which can be considered as a fusion of Prana and Agni. Minute audible nada can be considered as Shruti[1]. The difference in number of vibrations between two notes can also be considered as Shruti. The sound or nada which has a characteristic property of pleasing can be called Swara. We have seven notes or Saptaswaras namely Shadja (S), Rishabha (R), Gandhara (G), Madhyama (M), Panchama (P), Dhaivatha (D) and Nishada (N). S and P are unique and have no variations whereas R, G, M, D, N undergo variations. S and P are called PrakrutiSwaras and the others are known as VikrutiSwaras. It is a well-known fact that there exists 22 shrutis in Carnatic Music. Here is an attempt to explain the mathematical aspects of these Shrutis.

Concept of 22 SHRUTHIS

In the world of music, it is a universally accepted fact that there are seven main distinguishable notes or frequencies in an octave. In the Carnatic music context, these swaras are S, R, G, M, P, D and N,analogous to A, B,C,D,E,F and G in western music. Prakrutiswaras are those which are unique and have no variations – S &P.
Vikrutiswaras are the other 5 swaras which have variations – R, G, M, D and N. These can be compared with the flat and the sharp notes of the western music framework.

Positioning of Prakruti and VikrutiSwaras

According to the Seventy Two Melakarta System, the following are the different types of R, G, M, D and N:

R → Suddha, Chatushruti, Shatshruti
G → Suddha, Sadharana, Antara
M → Suddha, Prati
D → Suddha, Chatushruti, Shatshruti
N → Suddha, Kaisiki, Kakali

ChatushrutiRishabha and SuddhaGandhara occupy the same place or swarasthana. Similarly, ShatshritiRishabha and SadharanaGandhara occupy the same place. ChatushrutiDhaivatha and SuddhaNishada have the same place.ShatshrutiDhaivatha and KaisikiNishada have the same place.
This makes a total of 12 swarasthanas per octave. A raga in Carnatic music is defined as a melody obtained by a specific combination of these 12 notes in a particular order. Various combinations of these 12 notes arranged in the increasing order of their frequencies gives rise to what is known as the 72 Melakartha System. The Melakartha System is a numbered list of main or parent ragas in s specific order. There are many other smaller ragas derived from these main ragas, containing only some pf the swaras from their parent raga[2].
In most cases these 12 notes are sufficient to classify ragas using text or notations. However, what is observed is that there is a subtle difference between the same notes being sung in two different ragas. An example could be how the note chatushruthirishabha sung in Abhogi raga is audibly different from how it is sung in Kalyani, or how the note kaishikinishada sung in Madhyamavathi is different from the way it is sung in Kharaharapriya or Harikamboji. The way a note is sung with its ornamentation is called as gamakam. This gamakam of a note will sound different from the plain note, depending on the raga. The sliding, gliding patterns and gamakam of the same note can be different in different ragas. This means that each of these 12 notes can also have sub-divisions. In addition to this, it is also observed that there is a subtle difference in the way two singers approach the same note while singing a raga. This difference in frequencies is also a contributing factor to the concept of 22 shruthis[3]. This work attempts to represent these different variation in notes and to explain the underlying mathematical calculations.

Mathematical theory

We know that in a stringed instrument, the number of vibrations and length of the portion of the string are inversely proportional to each other. Suppose S to S- is the Shadja of upper octave and S is Shadja of middle octave and *l*is the distance between these two Shadjas andthe number of vibrations corresponding to length *l is n*,

then for $length = \frac{l}{2}$, the number of vibrations will be 2n, for $length = \frac{2}{3}l$, the number of vibrations will be $\frac{3}{2}n$, for $length = \frac{4}{5}l$, no. of vibrations will be $\frac{5}{4}n$, for $length = \frac{8}{9}l$, no.of vibrations will be $^{9}/_{8}n$, for length =*l*, no. ofvibrations will be *n.*

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓
S(1) R(9/8) G(5/4) M(4/3) P(3/2) D N S- R- G-

Figure 1. Swarasthanas and Ratios

For simplicity, let us consider that Madhya sthayi shadja has number of vibrations as *n*and the length of the string between S and S- is *l*. This would mean tara shadja has number of vibrations as 2n. In the figure above, the number of vibrations for each swarasthana have been indicated as a multiple of *n.*

It is known that when the number of vibrations are $\frac{9}{8}n$, $\frac{5}{4}n$, $\frac{4}{3}n$, $\frac{3}{2}n$ the Swarasthanas are *chat R*, *Antara G*, *Suddha M* and *P* respectively. Now we shall assess the values of *D*, *N* and *S-*. *P* is in the 5th position from *S*, so we get *D* at 5th position from *R*, *N* at the 5th place from *G*, *S-* at 5th position from *M*. $R = \frac{9}{8}n\ implies\ D = \frac{3}{2} \times \frac{9}{8}n = \frac{27}{16}n$ which corresponds to *chat D*. $G = \frac{5}{4}n\ implies\ N = \frac{3}{2} \times \frac{5}{4}n = \frac{15}{8}n$ which corresponds to *Kakali N*. $M = \frac{4}{3}n\ impliesS- = \frac{3}{2} \times \frac{4}{3}n = 2n.$

We have:

$$S(1)\ R\left(\frac{9}{8}\right)\ G\left(\frac{5}{4}\right)\ M\left(\frac{4}{3}\right)\ P\left(\frac{3}{2}\right)\ D\left(\frac{27}{16}\right)\ N\left(\frac{15}{8}\right)\ S-(2) \qquad (1)$$

This is the scale of Shankarabharana.

We also note that $S{:}P\ is\ 1{:}3/2$

$S{:}G\ is\ 1{:}5/4$

$S{:}M\ is\ 1{:}4/3$

$S{:}R\ is\ 1{:}9/8$

$S{:}S-\ is\ 1{:}2$

These combinations have melody and harmony when sung or played on instruments.

VikrutiBhedas: Let us consider (1). Considering R as the base and playing the same scale, we get the following values for VikrutiSwaras[4].

Original position	New position	Original ratio	New ratio w.r.t changed position
S	-	1	-
R	S	9/8	$\frac{9}{8} \div \frac{9}{8} = 1$
G	R	5/4	$\frac{5}{4} \div \frac{9}{8} = \frac{5}{4} \times \frac{8}{9} = \frac{10}{9}$
M	G	4/3	$\frac{4}{3} \times \frac{8}{9} = \frac{32}{27}$
P	M	3/2	$\frac{3}{2} \times \frac{8}{9} = \frac{4}{3}$
D	P	27/16	$\frac{27}{16} \times \frac{8}{9} = \frac{3}{2}$
N	D	15/8	$\frac{15}{8} \times \frac{8}{9} = \frac{5}{3}$
S-	N	2	$2 \times \frac{8}{9} = \frac{16}{9}$

Table 1. Vikruti Bhedas

All these values in Table 1 above, 10/9, 32/27, 4/3, 3/2, 5/3, 16/9 correspond to the scale of Kharaharapriya. Similarly, when G is the base, we get the values 1, 16/15, 6/5, 27/20, 3/2, 8/5, 9/5. When M is the base, we get the values 1, 9/8, 81/64, 45/32, 3/2, 27/16, 5/8. When P is the base, we get the values 1, 9/8, 5/4, 4/3, 3/2, 5/3, 16/9. When D is the base, we get the values 1, 10/9, 32/27, 4/3, 40/27, 128/81, 16/9. When N is the base, we get the values 1, 16/15, 6/5, 4/3, 64/45, 8/5, 9/5. These values can therefore be summarized as in the table 2 below.

Base swara (S)	**R**	**G**	**M**	**P**	**D**	**N**	**S-**
S	9/8=1.125	5/4=1.25	4/3=1.333	3/2=1.5	27/16=1.6875	15/8=1.875	2
R	10/9=1.111	32/27=1.851	4/3=1.333	3/2=1.5	5/3=1.666	16/9=1.777	2
G	16/15=1.066	6/5=1.2	27/20=1.35	3/2=1.5	8/5=1.6	9/5=1.8	2
M	9/8=1.125	81/64=1.2656	45/32=1.4062	3/2=1.5	27/16=1.6875	15/8=1.875	2
P	9/8=1.125	5/4=1.25	4/3=1.333	3/2=1.5	5/3=1.666	16/9=1.777	2
D	10/9=1.111	32/27=1.185	4/3=1.333	40/27=1.48	128/81=1.56	16/9=1.777	2
N	16/15=1.066	6/5=1.2	4/3=1.333	64/45=1.422	8/5=1.6	9/5=1.8	2

Table 2. Ratios of swarasthanas considering different base or starting swaras

VikrutiSwaras in ascending order will be as follows.

R	R1	16/15
	R2	10/9
	R3	9/8
G	G1	32/27
	G2	6/5
	G3	5/4
	G4	81/64
M	M1	4/3
	M2	27/20
	M3	45/32
	M4	64/45
P	P1	40/27
	P2	3/2
D	D1	128/81
	D2	8/5
	D3	5/3
	D4	27/16
N	N1	16/9
	N2	9/5
	N3	15/8

Table 3. Vikruthi Swaras in ascending order of values

In addition to the above values in Table 3, we have 3 more values from VadiSamvadiSystem. They are R1 = 256/243, M2 = 729/512 and N3 = 243/128. We make use of the following values of VikrutiSwaras in finding Twenty two or DwavimshatiShrutis (along with S = 1 and S- = 2)[5].

R	1R1	256/243
	2R1	16/15
	1R2	10/9
	2R2	9/8
G	1G2	32/27
	2G2	6/5
	1G3	5/4
	2G3	81/64
M	1M1	4/3
	2M1	27/20
	1M2	45/32
	2M2	64/45
	3M2	729/512
P	P1	40/27
	P2	3/2
D	1D1	128/81
	2D1	8/5
	1D2	5/3
	2D2	27/16
N	1N2	16/9
	2N2	9/5
	1N3	15/8
	2N3	243/128

Table 4. List of Swarasthanas

Poorna Shruti

The ratio 256/243 is called Poorna Shruti and the following seven pairs of notes have that ratio.
(S, 1R1) = (1, 256/243) = 256/243
(2R2, 1G2) = (9/8, 32/27) = 256/243
(2G3, 1M1) = (81/64, 4/3) = 256/243
(3M2, P2) = (729/512, 3/2) = 256/243
(P2, 1D1) = (3/2, 128/81) = 256/243
(2D2, 1N2) = (27/16, 16/9) = 256/243
(2N3, S-) = (243/128, 2) = 256/243

Pramana Shruti

The ratio 81/80 is called PramanaShruti. The following ten pairs have the ratio 81/80.
(1R1, 2R1) = (256/243, 16/15) = 81/80
(1R2, 2R2) = (10/9, 9/8) = 81/80
(1G2, 2G2) = (32/27, 4/5) = 81/80
(1G3, 2G3) = (5/4, 81/64) = 81/80
(1M1, 2M1) = (4/3, 27/20) = 81/80
(1M2, 3M2) = (45/32, 729/512) = 81/80
(1D1, 2D1) = (128/81, 8/5) = 81/80
(1D2, 2D2) = (5/3, 27/16) = 81/80
(1N2, 2N2) = (16/9, 9/5) = 81/80
(1N3, 2N3) = (15/8, 243/128) = 81/80

Nyuna Shruti

The ratio 25/24 is called NyunaShruti and the following five pairs of notes have the ratio 25/24.
(2R1, 1R2) = (16/15, 10/9) = 25/24
(2G2, 1G3) = (6/5, 5/4) = 25/24
(2M1, 1M2) = (27/20, 45/32) = 25/24
(2D1, 1D2) = (8/5, 5/3) = 25/24
(2N2, 1N3) = (9/5, 15/8) = 25/24

Results And Discussions

Every note on swara vibrates at a certain frequency. The difference in these vibrations between different swaras is called shruthi. The complete list of swarasthanas along with their ratios is as shown in Table 4. Swaraantaras or distance between notes will give 22 shrutis**.** In other words,7 Poorna shruti related values, 10Pramana shruti related values and 5Nyuna shruti related values give the required 22 or dwavimshati shrutis[6].

Conclusion

Although it is universally accepted that there are 12 perceivably distinct swaras in a musical octave, the system of Carnatic music is able to identify certain sub-divisions of these notes. This can be attributed to two different reasons:

1. The same note has a different sound or gamaka or way of singing in different ragas:
2. The same note when sung by different singers singing the same raga isobserved to have a difference in frequency or the way it sounds.

This suggests that there are sub-divisions for most of these 12 swarasthanas. These variations and the difference in ratios of all their combinations give rise to 22 shruthis. This paper has highlighted the concept pf these shruthis and has explained the mathematical framework.

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