BIANCHI TYPE VI_0 BULK VISCOUS STRING COSMOLOGICAL MODEL WITH SPECIAL FORM OF DECELERATION PARAMETER IN $f(R,T)$ GRAVITY

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ABSTRACT

In this paper, we derive field equations of $f(R,T)$ gravity with the help of a spatially homogenous and anisotropic Bianchi type VI_0 metric in presence of bulk viscous fluid, containing one-dimensional cosmic strings. To obtained the determinate solution, a spatial form of deceleration recently proposed Singha and Debnath for FRW metric is used. We have also used the barotropic equation of state for density and the pressure and bulk viscous pressure is assumed to be proportional to energy density. Some physical and Kinematical properties of the model are also discussed.

Keywords: *Bianchi type model VI_0, Bulk model, String model, $f(R,T)$, Special form of deceleration parameter.*

1 Introduction

Harko et al. (2011) developed $f(R,T)$ modified theory of gravity, where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and the trace T of the energy-momentum tensor. It is to be noted that the dependence of T may be induced by exotic imperfect fluid or quantum effects. they have obtained the gravitational field equations in the metric formalism, as well as, the equations of motion of test particles, which follows from the covariant divergence of the stress-energy tensor. They have derived some particular models corresponding to specific choices of function $f(R,T)$. They have demonstrated the possibility of reconstruction of arbitrary FRW cosmologies by an appropriate choice of the function $f(R,T)$.

In $f(R,T)$ gravity, the field equations are obtained from a variational, Hilbert-Einstein type, principle.

The action principle for this modified theory $f(R,T)$ gravity is given by

$$S=\frac{1}{16\pi G}\int f(R,T)\sqrt{-g}\,d^4x+\int L_m\sqrt{-g}\,d^4x \quad (1.1)$$

Where $f(R,T)$ is an arbitrary function of the Ricci scalar R, T is the trace of stress energy tensor of matter, T_{ij} and L_m is the matter Lagrangian density.

We define the stress energy tensor of matter as

$$T_{ij}=\frac{-2}{\sqrt{-g}}\frac{\partial(\sqrt{-g})}{\partial g^{ij}}L_m, \quad (1.2)$$

And it's trace by $T=g^{ij}T_{ij}$ respectively. BY assuming that L_m of matter depends only on the metric tensor components g_{ij}, and not on it's derivatively, we obtain

$$T_{ij}=g_{ij}L_m-2\frac{\partial L_m}{\partial g^{ij}} \quad (1.3)$$

By varying the action principle (1.1) with respect to metric tensor, the corresponding field equations of $f(R,T)$ gravity are obtained as

$$f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}+(g_{ij}\nabla^i\nabla_i-\nabla_i\nabla_j)f_R(R,T)$$
$$=8\pi T_{ij}-f_T(R,T)T_{ij}-f_T(R,T)\Theta_{ij} \quad (1.4)$$

Where

$$\Theta_{ij}=-2T_{ij}+g_{ij}L_m-2g^{lk}\frac{\partial^2 L_m}{\partial g^{ij}\partial g^{lm}} \quad (1.5)$$

Here

$$f_R=\frac{\partial f(R,T)}{\partial R},\quad f_T=\frac{\partial f(R,T)}{\partial T}$$

Here ∇_i is the covariant derivation and T_{ij} is standard matter energy-momentum tensor derived from the Lagrangian L_m.

It can be observed that when $f(R,T)=f(R)$, then (1.4) yield the field equations of $f(R)$ gravity.

The problem of the perfect fluids described by an energy density ρ, pressure p and four velocity u^i is complicated. Since there is no unique definition of the matter Lagrangian. However, here, we assume that the stress energy tensor of the matter is given by

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij} \quad (1.6)$$

And the matter Lagrangian can be taken as $L_m = -p$ and we have

$$u^i \nabla_j u_i = 0, \quad u^i u_i = 1 \quad (1.7)$$

Then with the use of equ. (5) we obtain for the variation of stress-energy of perfect fluid the expression

$$\Theta_{ij} = -2T_{ij} - p g_{ij} \quad (1.8)$$

It is mentioned here that these field equations depend on physical nature of the matter field. Many theoretical models corresponding to different matter contributions for $f(R,T)$ gravity are possible. However, Harko et. al. gave three classes of these models

$$f(R,T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T) \end{cases}$$

Assuming,

$$f(R,T) = R + 2f(T) \quad (1.9)$$

as a first choice, where $f(T)$ is an arbitrary function of trace of the stress energy tensor of matter

Then from (1.3) and (1.4), we get the gravitational field equation as

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} - 2f'(T) T_{ij} - 2f'(T)\Theta_{ij} + f(T) g_{ij} \quad (1.10)$$

Where the overhead prime indicates differentiation with respect to the argument .

If the matter source is a perfect fluid,

$$\Theta_{ij} = -2T_{ij} - p g_{ij}$$

Then the field equations become

$$R_{ij} - \frac{1}{2} R g_{ij} = 8\pi T_{ij} + 2f'(T) T_{ij} + \left[2pf'(T) + f(T)\right] g_{ij} \quad (1.11)$$

At the early stages of the evolution of Universe, in general , it is spatially homogenous and anisotropic. Bianchi spaces are useful tools for constructing spatially homogenous and anisotropic cosmological models in general relativity and scalar-tensor theories of gravitation. Reddy et. al. (2012a, 2012b) have obtained Kaluza- Klein cosmological model in the presence of perfect fluid source and Bianchi type III cosmological model in $f(R,T)$ gravity using the assumption of law of variation for the Hubble parameter proposed by Bermann (1983), Adhav (2012) has obtained LRS Bianchi type-I cosmological model in $f(R,T)$ gravity using the same assumption of law of variation for the Hubble parameter proposed by Bermann (1983). Shamir et al. obtained exact solution of Bianchi type-I and type-V cosmological model in $f(R,T)$ gravity. Chaubey and Shukla (2013) have obtained a new class of Bianchi cosmological models in $f(R,T)$ gravity. Reddy and Santi Kumar (2013) have presented some anisotropic cosmological models in this theory. Rao and Neelima (2013) have discussed perfect fluid Einstein-Rosen universe in $f(R,T)$ gravity. Recently Rao and Neelima (2013) have obtained perfect fluid Bianchi type VI0 perfect fluid model in $f(R,T)$ gravity.

At the time of particle creation in the early universe and during formation of galaxies (Hu 1983), when neutrinos decoupled from cosmic fluid (Misner 1968) viscosity arises. Many authors have studied bulk viscous cosmological models in general relativity. Johri and Sudarsan have investigated bulk viscous cosmological model in Brans-Dicke theory of gravitation. Bulk viscous cosmological model in Saez-Ballester theory of gravitation have been discussed be

several authors. Recently Rao et al., Reddy et al., Naidu et al. have discussed the bulk viscous cosmological models in modified theory of gravity proposed by Harko et. al (2011). Very recently Naidu et al. investigated Bianchi type V Bulk viscous string cosmological model in f(R,T) gravity.

In this paper, we have studied Bianchi type VI_0 string Cosmological model of the early universe with bulk viscosity in modified theory of $f\ (R,T)$ gravity with special form of deceleration parameter. By using the barotropic equation of state and bulk viscous pressure is assumed to be proportional to energy density, solutions of field equations obtained. We have also discussed the physical and kinematical properties of the models.

2 Metric and field equations

We consider the spatial homogenous and anisotropic space-time represented by the Bianchi type VI_0 metric as

$$ds^2 = -dt^2 + a_1{}^2 dx^2 + a_2{}^2 e^{-2x} dy^2 + a_3{}^2 e^{2x} dz^2 \quad (2.1)$$

Where a_1, a_2, a_3 are functions of cosmic time t .

We consider the energy momentum tensor for bulk viscous fluid containing one dimensional cosmic strings as

$$T_{ij} = (\rho + p)u_i u_j + p g_{ij} - \lambda x_i x_j \qquad (2.2)$$

Here we take

$$\bar{p} = p - 3\zeta H \qquad (2.3)$$

Where ρ is the rest energy density of system, $\zeta(t)$ is the coefficient of the bulk viscosity, $3\zeta H$ is usually known as bulk viscous pressure, H is the Hubble's parameter, u^i is the four velocity of the fluid, x^i is the direction of the string and λ is the string tension density. Also $u^i = \delta_4^i$ is the four velocity vector which satisfies

$$g_{ij}u^i u_j = -x^i x_j = -1 \text{ and } u^i x_j = 0 \qquad (2.4)$$

And $T_1^1 = \bar{p} - \lambda$, $T_2^2 = \bar{p}$, $T_3^3 = \bar{p}$, $T_4^4 = \rho$

$$T = T_1^1 + T_2^2 + T_3^3 + T_4^4 = (3\bar{p} - \rho - \lambda) \qquad (2.5)$$

Here we also consider $\bar{p}, \rho$ and λ as functions of time t only.

Using co-moving coordinates and Eqs. (2.2) – (2.5), the $f(R,T)$ gravity field equation (2.1) with proper choice of the function (Harko et al. 2011)

$f(T) = \mu T$ where μ is constant. (2.6)

For metric (2.1) , field equation yield the form

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_3}{a_3} + \frac{1}{a_1{}^2} = \bar{p}(8\pi + 7\mu) - \lambda(8\pi + 3\mu) - \mu\rho \qquad (2.7)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_3}{a_3} - \frac{1}{a_1{}^2} = \bar{p}(8\pi + 7\mu) - \lambda\mu - \mu\rho \qquad (2.8)$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} - \frac{1}{a_1{}^2} = \bar{p}(8\pi + 7\mu) - \lambda\mu - \mu\rho \qquad (2.9)$$

$$\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_3}{a_3} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} - \frac{1}{a_1{}^2} = -\rho(8\pi + 3\mu) + \lambda\mu + 5\mu\bar{p} \qquad (2.10)$$

$$\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_2}{a_2} = 0 \qquad (2.11)$$

where an overhead dot represent differentiation with respect to t.

We define the following parameters for the metric (2.1) which will be used in solving the above field equations.

The spatial volume and scale factor are given by

$$V^3 = a_1 a_2 a_3 \qquad (2.12)$$

$$a = (a_1 a_2 a_3)^{\frac{1}{3}} \qquad (2.13)$$

The physical quantities of observational interest in cosmology are the expansion scalar θ, the mean anisotropy parameter A_h and shear scalar σ^2 which are defined as

$$\theta = u^i_{;j} = \frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3} \qquad (2.14)$$

$$A_h = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2,$$

$$\Delta H_i = H_i - H, \quad i = 1,2,3 \qquad (2.15)$$

$$\sigma^2 = \sigma^{ij}\sigma_{ij} = \frac{1}{3}\left[\left(\frac{\dot{a}_1}{a_1}\right)^2 + \left(\frac{\dot{a}_2}{a_2}\right)^2 + \left(\frac{\dot{a}_3}{a_3}\right)^2 - \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_1}{a_1}\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3}\right] \qquad (2.16)$$

3 Solutions and the model

On integrating equation .(2.11), we obtained

$$a_3 = k a_2 \qquad (3.1)$$

Where k is a constant of integration, which can be chosen as unity without loss of any generality, so that we have

$$a_3 = a_2 \qquad (3.2)$$

The field Equations. (2.7) – (2.10) reduces to the following independent equations

$$\frac{\ddot{a}_2}{a_2} - \frac{\ddot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1}\frac{\dot{a}_3}{a_3} + \frac{2}{a_1{}^2} = -\lambda(8\pi + 2\mu) \qquad (3.3)$$

$$\frac{\ddot{a}_3}{a_3} - \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_3}{a_3} - \frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} = 0 \qquad (3.4)$$

$$\frac{\dot{a}_1}{a_1}\frac{\dot{a}_2}{a_2} + \frac{\dot{a}_1}{a_1}\frac{\dot{a}_3}{a_3} + \frac{\dot{a}_2}{a_2}\frac{\dot{a}_3}{a_3} - \frac{1}{a_1{}^2} = -\rho(8\pi + 3\mu) + \lambda\mu + 5\mu\bar{p} \qquad (3.5)$$

Now equations. (3.2) – (3.5) are a system of four independent equations in six unknowns, a_1, a_2, a_3, p, ρ and λ. Also the equations are highly non-linear. Hence to find a determinate solution we use the following physically plausible conditions:

i) Spatial form of deceleration parameter as

$$q = -a\frac{\ddot{a}}{\dot{a}^2} = -1 + \frac{k}{1+a^k} \quad (3.6)$$

Where a is mean scale factor of the universe, $k > 0$ is constant. This law has been recently proposed by Singha and Debnath for FRW metric.

We know that the universe has

i)decelerating expansion if $q > 0$

ii)an expansion with constant rate if $q = 0$

iii)accelerating power law expansion if $-1 < q < 0$

iv)exponential expansion (or deSitter expansion) if $q = -1$

v)super-exponential expansion if $q < -1$,

From (3.6) we obtain the Hubble parameter as

$$H = \frac{\dot{a}}{a} = m(1 + a^{-k}) \quad (3.7)$$

Where m is an integration constant.

Now by integrating above equation, one can obtain the mean scale factor as

$$a = (e^{mkt} - 1)^{\frac{1}{k}} \quad (3.8)$$

Choosing $m = 1$, the mean scale factor becomes

$$a = (e^{kt} - 1)^{\frac{1}{k}} \quad (3.9)$$

ii) The shear scalar σ^2 is proportional to scalar expansion θ so that we can take Collins et al. (1980)

$$A = B^n \quad (3.10)$$

iii) For a barotropic fluid, the combined effect of the proper pressure and the bulk viscous pressure can be expressed as

$$\bar{p} = p - 3\zeta H = \varepsilon\rho \quad (3.11)$$

Where

$$\varepsilon = \varepsilon_0 - \beta \quad (0 \le \varepsilon_0 \le 1), \; p = \varepsilon_0 \rho \quad (3.12)$$

And ε_0 and β are constants.

Now from Eqs. (2.13), (3.2), (3.9) and (3.10), we obtain

$$\left.\begin{aligned} a_1 &= (e^{kt} - 1)^{\frac{3n}{(n+2)(1+q)}} \\ a_2 = a_3 &= (e^{kt} - 1)^{\frac{3}{(n+2)(1+q)}} \end{aligned}\right\} \quad (3.13)$$

Using equations (3.13) and the metric (2.1) can be written as

$$ds^2 = -dt^2 + (e^{kt} - 1)^{\frac{6n}{(n+2)(1+q)}} dx^2 + (e^{kt} - 1)^{\frac{6}{(n+2)(1+q)}} \left[e^{-2x} dy^2 + e^{2x} dz^2\right] \quad (3.14)$$

4 Physical properties of the model

Equation (3.14) represents the bulk viscous string Bianchi type-VI_0 cosmological model in the $f(R,T)$ modified theory of gravitation which is physical significant for the study of early stage of universe. The physical and kinematical parameter of the model, which are important for the discussion of cosmological model (3.12) are the following.

Spatial volume

$$V^3 = (e^{kt} - 1)^{\frac{3}{k}} \quad (4.1)$$

Scalar of expansion

$$\theta = \frac{3}{(1 - e^{-kt})} \quad (4.2)$$

The mean Hubble parameter

$$H = \frac{1}{(1 - e^{-kt})} \quad (4.3)$$

The deceleration parameter

$$q = -1 + ke^{-kt} \quad (4.4)$$

The average anisotropy parameter,

$$A_h = \frac{2(n-1)^2}{(m+2)^2} \quad (4.5)$$

Shear scalor,

$$\sigma^2 = \frac{3(n-1)^2}{(n+2)^2(1 - e^{-kt})^2} \quad (4.6)$$

The string tension density,

$$\lambda = \frac{-1}{(8\pi + 2\mu)}\left\{\left[\frac{3(n-1)ke^{-kt}}{(n+2)(1-e^{-kt})^2} + 2(e^{kt} - 1)^{\frac{-6n}{(n+2)k}}\right]\right\} \quad (4.7)$$

The energy density,

$$\rho = \frac{-1}{[8\pi + \mu(3-5\in)]}\left\{ \begin{array}{l} \frac{\mu}{(8\pi+2\mu)}\left[\frac{3(n-1)ke^{-kt}}{(n+2)(1-e^{-kt})^2} + 2(e^{kt}-1)^{\frac{-6n}{(n+2)k}}\right] \\ + \frac{9(2n+1)}{(n+2)^2(1-e^{-kt})^2} - (e^{kt}-1)^{\frac{-6n}{(n+2)k}} \end{array} \right\} \quad (4.8)$$

The isotropic pressure,

$$p = \frac{-\in_0}{[8\pi + \mu(3-5\in)]}\left\{ \begin{array}{l} \frac{\mu}{(8\pi+2\mu)}\left[\frac{3(n-1)ke^{-kt}}{(n+2)(1-e^{-kt})^2} + 2(e^{kt}-1)^{\frac{-6n}{(n+2)k}}\right] \\ + \frac{9(2n+1)}{(n+2)^2(1-e^{-kt})^2} - (e^{kt}-1)^{\frac{-6n}{(n+2)k}} \end{array} \right\} \quad (4.9)$$

The coefficient of bulk viscosity,

$$\zeta = \frac{\in - \in_0}{3[8\pi + \mu(3-5\in)]}\left\{ \begin{array}{l} \frac{\mu}{(8\pi+2\mu)}\left[\frac{3(n-1)ke^{-kt}}{(n+2)(1-e^{-kt})^2} + 2e^{-kt}(e^{kt}-1)^{1-\frac{6n}{(n+2)k}}\right] \\ + \frac{9(2n+1)}{(n+2)^2(1-e^{-kt})^2} - e^{-kt}(e^{kt}-1)^{1-\frac{6n}{(n+2)k}} \end{array} \right\} \quad (4.10)$$

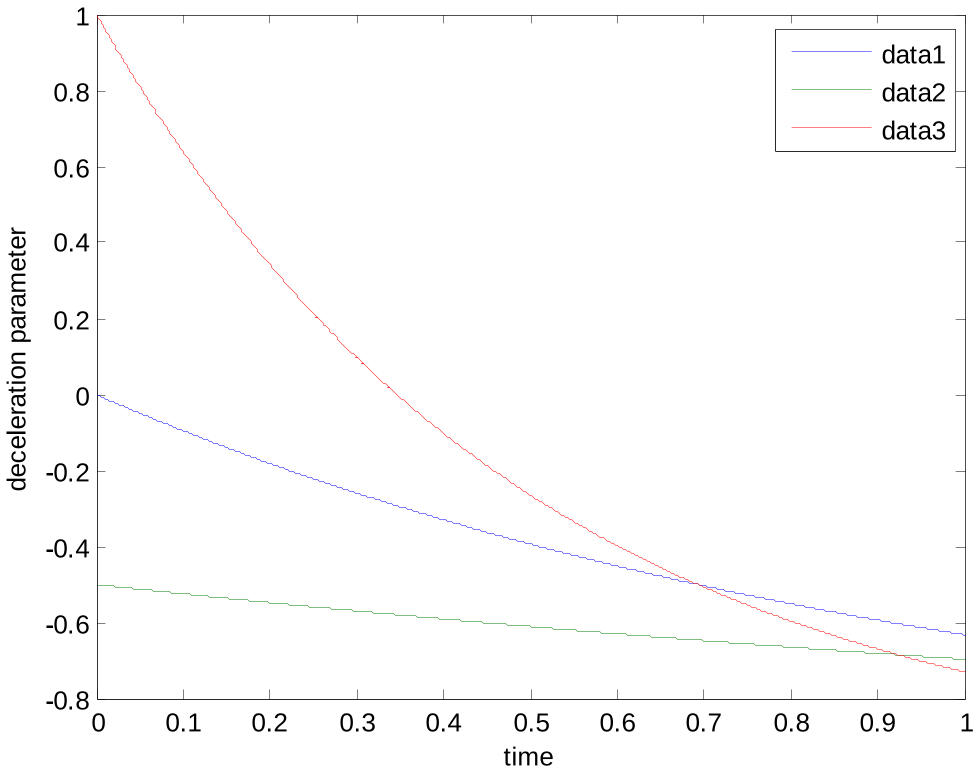

Fig. The variation of q vs t

Fig shows the variation of deceleration parmeter vs cosmic time t, for data1 $k=1$, for data 2 $k=.5$ for data3 $k=2$

The above results are useful to discuss the behavior of $f(R,T)$ gravity cosmological model given by (3.14). The results (3.14) possess initial singularity of the point. Also, it can be observed that the physical and kinematical parameters $\rho, p, H, \theta, \sigma^2$ and λ approach infinity as $t=0$ and approach to zero as $t \to \infty$,since $k>0$. The results (4.4) implies that the universe is expanding, super exponentially if $k<1$, with constant rate if $k=1$, acceleration expanding if as $k=1$ as $t=0$. The universe is expanding exponentially as $t \to \infty$. The results (4.10) implies that parameter ζ initially approaches to infinity, for $k > \frac{6n}{n+2}$. It can also be observed that the model is expanding with time. From Equ. (4.2) and (4.6), it can also be observed that the model in this theory becomes anisotropic except $n=1$. Also, it can be observed that for $n=1$, the model is isotropic and shear free.

5 Conclusions

In this paper, we have presented spatially homogeneous and anisotropic Bianchi type VI_0 cosmological model in the presence of bulk viscous fluid with spatially form of deceleration parameter & one dimensional cosmic strings in $f(R,T)$ gravity formulated by Harko et. al. (2011) by modifying general relativity to explain the challenging problem of late time acceleration of the universe. We have found a determinate solution of the highly non-linear field equations of this theory, by using i) special form of deceleration parameter recently proposed by Singha and Debnath ii) a barotropic equation of state for fluid for pressure and energy density iii) proportionality of the bulk viscous pressure and the energy density. It is observed that the model have initial singularity. Also It is observed that initially, the universe is expanding, super exponentially if $k<1$, with constant rate if $k=1$, acceleration expanding if $k=1$. The universe is expanding exponentially for large value for of cosmic time t. It is also observed that the bulk viscosity decrease with increase in cosmic time which leads to the inflationary model for $k > \frac{6n}{n+2}$..

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