EPQ MODEL WITH INVENTORY DEPENDENT RATE PARAMETER

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ABSTRACT

This work presents EPQ model for deteriorating items. The demand pattern used is mix type. Here demand rate is inventory dependent when production is in process and assumes it constant after maximum inventory level reaches. The optimum solution of the model is derived by using simple differential calculus method. The effect of rate at which inventory get consumed is discussed in this model. In this model the total cost function shows the convexity. Mathematical Model gets verified by using numerical example. Sensitivity analysis had been carried out.

Keywords:- *EPQ, Inventory dependent*

1. Introduction

The Economic Production Quantity models are extensively used to control inventories due to its ease in application. Traditional models assume that only perfect quality products produced during production and demand rate is also constant. .But actually it is not possible. When product is in maturity stage then constant demand may possible. Buying capacity of the customer increases if the stock is in bulk amount. In perishable items deterioration is the common phenomenon. Numerous research had been carried out by number of researchers by considering different rate of demand and deterioration. Rosenblatt and Lee [1] studied imperfect production process with different rates of deterioration. Teng *et.al* [2] developed the EOQ model for deteriorating items by considering stock dependent demand. Their investigation shows that optimal solution is highly sensitive to demand and selling price. Lin and Gong [3] presented EPQ model under random machine breakdown and consider corrective maintenance period as fixed. Teng *et.al* [4] assumes that the demand function fluctuating with time and purchase cost as positive. Lin and Gong [5] used LIFO policy to investigate the effect of imperfect production process. Jain and Sharma [6] studied inventory model for price and stock dependent demand. Widyadana and Wee [7] developed EPQ deteriorating model with machine breakdown and stochastic repair time. Cheng *et.al*.[8] examined the effect of random machine breakdown on optimal run time and average cost with constant demand and backorder. Chiu [9] considers the effect of reworking on the EPQ model with backlogging. Chung et. al. [10] developed the EPQ model for deteriorating items with random machine unavailability and shortage accrued from it. Mishra and Singh [11] established inventory model with power form stock dependent demand and cubic deterioration. Shah and Patel [12] suggested the price sensitive stock dependent demand to construct the inventory model with two credit policies and price negotiation. Krishnamoorthi and Panayappan[13] considered the regular rework of defective items to convert them into finished goods and presented the defect sales return. It determines the optimal production lot size. Garg et.al. [14] presented an EPQ model for ramp type production and demand rate and considering deterioration as non-instantaneous and Weibull in nature. Khedlekar [15] attempted to establish an exponential demand with disrupted production system. Min et.al [16] assumes demand rate to be dependent on the retailers current inventory level and developed a replenishment model with trade credit for deteriorating items. From the literature review, it is observed that same demand pattern had been discussed by number of researcher on different time period. Here in present study we used inventory dependent demand when production is in process and constant demand when sufficient stock get build up. This paper is in four sections. Literature is in section 1. Mathematical model development is discussed in section 2. Section 3 has numerical analysis and finally concluded in section 4.

2. Model development

Assumptions:-

Assumptions made for the development of the model is as follows.

a) Constant production rate
b) Demand rate is less than production rate.
c) Inventory level dependent in production time and constant after production stops.
d) Constant rate of deterioration.
e) The cost of deteriorated items is considered as constant.
f) Constant Inventory holding cost
g) Deterioration of the items start as it enters into inventory.
h) Shortages are not allowed.

Notation:

I_1 – Inventory level in production time.
I_2 – Inventory level in no production time.
T_1– Production time.
T_2 – No Production time.
P –Production rate.
D – Basic demand rate.
θ – Deterioration rate.
α- Inventory dependent consumption rate parameter.
h – Holding cost.
C_d – Deterioration cost.
T – Production cycle time.
TC – Total cost.
TCT – Total cost per unit time.

The production will start at time t = 0. During the time period (0, T_1)the inventory will gradually build up and demand assumed is inventory dependent. Maximum inventory will be at time t=T_1. Later, production stops and available inventory is used to fulfill the demand. The deterioration start when item produced became the part of inventory. During no production time, demand assume as constant. Production system can be described by the following differential equations.

i) $$\frac{dI_1(t)}{dt} = P - D - \alpha I_1(t) - \theta I_1(t)$$
$0 \le t \le T_1$ (1)

j) $$\frac{dI_2(t)}{dt} = \text{-D-,}\theta I_1(t)$$
$0 \le t \le T_2$ (2)

k) With Initial boundary conditions $I_1(0) = 0$ and $I_2(T_2) = 0$, the solution to above equations is as follows.

l) $$I_1(t) = \frac{P-D}{\alpha+\theta}\left[1 - e^{-(\alpha+\theta)t}\right]$$
$0 \le t \le T_1$ (3)

m) $$I_2(t) = \frac{D}{\theta}\left[e^{\theta(T_2 - t)} - 1\right] \quad 0 \le t \le T_2 \qquad (4)$$

n) By using boundary condition $I_1(T_1) = I_2(0)$

o) $$\frac{P-D}{\alpha+\theta}\left[1 - e^{-(\alpha+\theta)T_1}\right] = \frac{D}{\theta}\left[e^{\theta(T_2)} - 1\right]$$

p) $$T_2 = \frac{P-D}{D}\left[T_1 - \frac{(\alpha+\theta)T_1^2}{2}\right] \qquad (5)$$

q) Total inventory is sum of inventory during production time and in no production time.

r) $$TI = \int_0^{T_1} I_1(t)dt + \int_0^{T_2} I_2(t)dt$$

s) The approximate total inventory can be expressed as follows,

t) $$TI = \frac{(P-D)T_1^2}{2} + \frac{DT_2^2}{2} \qquad (6)$$

u) Total number of items deteriorated can be formulated as follows.

v) $$TD = 0.5(P-D)(\alpha+\theta)T_1^3 - \alpha\left(\frac{(P-D)T_1^2}{2}\right) \qquad (7)$$

w) As all produced items get inspected so total inspection cost is

x) $$IC = c_i\left(\frac{(P-D)T_1^2}{2}\right)$$

y) Total cost is sum of Set up cost, Holding cost, Inspection cost and Deteriorating cost.

z)

aa) TC = A+ h(TI) + IC + C_d(TD)

bb) $$TC = A + h\left[\frac{(P-D)T_1^2}{2} + \frac{DT_2^2}{2}\right] + c_i\left(\frac{(P-D)T_1^2}{2}\right) + C_d\left[0.5(P-D)(\alpha+\theta)T_1^3 - \alpha\left(\frac{(P-D)T_1^2}{2}\right)\right] \quad (8)$$

cc) $TCT = \frac{TC}{T}$

dd) $T = T_1 + T_2$

$$TCT = \frac{TC = A + h\left[\frac{(P-D)T_1^2}{2} + \frac{DT_2^2}{2}\right] + c_i\left(\frac{(P-D)T_1^2}{2}\right) + C_d\left[0.5(P-D)(\alpha+\theta)T_1^3 - \alpha\left(\frac{(P-D)T_1^2}{2}\right)\right]}{T_1 + T_2} \quad (9)$$

The optimum production up time can be derived by satisfying the equation (10).

$$\frac{dTCT}{dT_1} = 0 \quad (10)$$

3. Numerical examples and sensitivity analysis

For the validation of theoretical aspects numerical example and sensitivity analysis has been carried out. The numerical data is adopted from Min *et al.*(16) .

A is set up cost and considered as Rs.30 per production cycle, C_d = Rs.5 /unit /unit time, C_i = Rs 2 /unit / unit time, H = Rs.2 / unit , unit time, P = 2500 units / unit time, D = 1200 units / unit time, $\alpha = 0.5$, $\theta = 0.1$

Convexity of total cost function is shown in fig. 1. The optimum value of T_1 is 0.124. The optimum total cost per unit time is TCT = Rs.231.83. Sensitivity analysis is carried out by changing each parameter by –40% to +40%, taking one parameter at a time and keeping others unchanged.

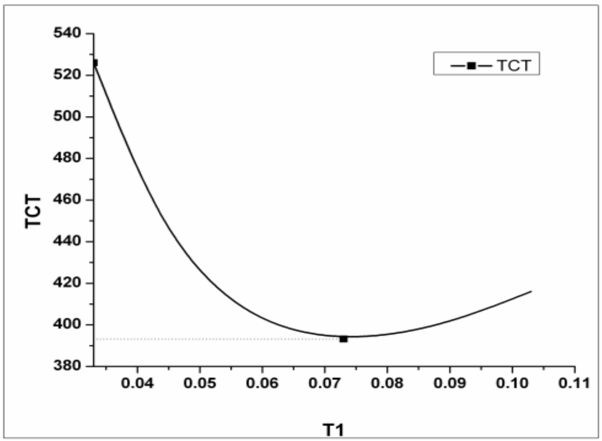

Fig.1: T_1 v/s TCT

Table No.1

A	HC	T_1	T_2
0.3	30.75	0.108	0.114
0.4	34.76	0.115	0.121
0.5	39.95	0.124	0.127
0.6	47.56	0.136	0.140
0.7	58.49	0.152	0.154

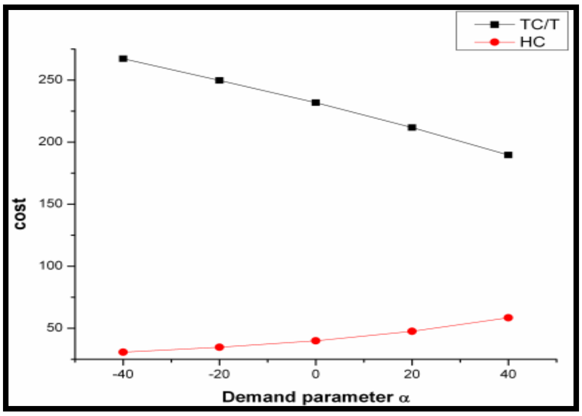

Fig.2 Demand parameters α v/s Cost

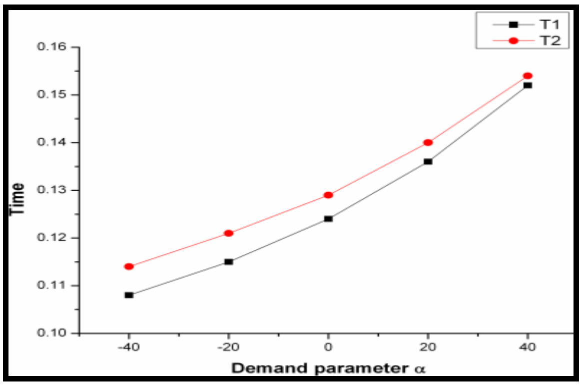

Fig.3 Demand parameters α v/s Time

Fig.2 shows that increase in demand parameter increases holding cost. But at the same time total cost goes on decreasing with increase in demand parameter. As demand is inventory dependent so demand will increase as the inventory level will increase. Fig. 3 shows that production time is highly sensitive to demand parameter. Production time increases due to increase in demand parameter. It means increase in inventory dependent consumption rate parameter increase demand. This attracts the attention of inventory managers to estimate the accurate value of the inventory dependent consumption rate parameter.

4. Conclusion

In this study, theoretical EPQ model has been studied for deteriorating items. Some interesting observations are presented. During inventory build up time, demand used is inventory level dependent and during inventory depletion time constant demand. Increase in demand parameter α decreases the TCT though increases the holding cost. Increase in consumption rate parameter decreases the total cost and holding cost. So proper selection of this parameter is important in decision making.

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