

A Mathematical Framework for Convex Optimization of Solar Energy Scheduling and Carbon Reduction in Educational Buildings

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Abstract

This paper develops a transparent convex optimization model for shifting flexible electricity demand toward available rooftop photovoltaic generation in educational buildings. Hourly grid import is represented by the positive part of demand minus solar generation. The objective combines time-dependent grid-emission factors with a quadratic import-balancing penalty, while operational constraints preserve total flexible demand and bound permissible shifting. Convexity guarantees global optimality. For a two-period reduction, the solution is obtained analytically by evaluating active-region stationary points, endpoints, and solar-saturation breakpoints. In an illustrative assumed-data example, the optimal shift is 4.75 kWh; grid imports change from (6,0) to (1.25,2.75) kWh and modeled operational emissions decrease from 4.80 to 2.375 kg CO₂e, a 50.52% reduction. These values are hypothetical and must not be interpreted as measured campus savings. The framework provides a reproducible mathematical baseline that can be extended with measured load, photovoltaic, tariff, comfort, storage, and marginal-emission data.

Keywords: convex optimization; photovoltaic self-consumption; demand response; carbon-aware scheduling; educational buildings; load shifting

Introduction

Educational buildings contain electricity-consuming activities whose timing may be partly flexible, including water pumping, equipment charging, laboratory preparation, and noncritical computing. At the same time, rooftop photovoltaic (PV) production and the carbon intensity of grid electricity can vary through the day. Demand-side management can improve the temporal match between building demand and local PV production (Fernández Bandera, Porsani, and Fernández-Vigil Iglesias 2023; Rehman, Khan, and Javaid 2021), while carbon-aware scheduling can explicitly use time-dependent emissions signals rather than energy quantity alone (Chen et al. 2024; Gou, Xu, and Sun 2025).

A schedule that maximizes PV self-consumption does not necessarily minimize operational carbon emissions when grid carbon intensity differs across periods. This motivates an optimization objective that makes the trade-off explicit. The mathematical structure adopted here follows standard convex-optimization principles (Boyd and Vandenberghe 2004). Unlike large simulation or mixed-integer energy-management systems, the present contribution deliberately seeks an analytically transparent baseline.

Research gap and contribution

Recent building-energy research increasingly combines flexible demand, PV, storage, model-predictive control, and carbon signals (Rehman, Khan, and Javaid 2021; Id Omar, Lê-Agopyan, and Sossan 2026; Li et al. 2026). Such models can be powerful but may obscure the fundamental scheduling rule or require extensive data. This study therefore contributes: (i) a compact convex formulation for flexible-load allocation under PV availability and time-varying emission factors; (ii) a proof of global optimality; (iii) an exact two-period solution; (iv) a reproducible illustrative calculation; and (v) a sensitivity interpretation that identifies how the optimal shift changes with the carbon-intensity difference and balancing weight.

Related Literature

PV self-consumption can be increased through demand-side management and flexible building loads (Fernández Bandera, Porsani, and Fernández-Vigil Iglesias 2023; Rehman, Khan, and Javaid 2021). Model-predictive and scheduling approaches have also coordinated storage, tariffs, peak shaving, and local PV use (Id Omar, Lê-Agopyan, and Sossan 2026). More recent research emphasizes carbon-aware operation, where temporal carbon signals become explicit optimization inputs (Chen et al. 2024; Gou, Xu, and Sun 2025). Building controls can additionally exploit HVAC flexibility to improve on-site PV matching (Li et al. 2026). These

studies motivate the present formulation but differ from its principal aim: an analytically solvable, auditable mathematical baseline suitable for educational-building planning before detailed field data are available.

Position of the proposed framework relative to representative approaches.

Approach	Main signal	Typical method	Relation to this study
PV self-consumption	PV/load matching	DSM/MPC	Motivates temporal load shifting
Building-to-grid control	Cost/grid services	MPC/optimization	Shows value of coordinated flexibility
Carbon-aware dispatch	Dynamic carbon intensity	OPF/MILP/robust optimization	Motivates time-dependent emission factors
Present model	PV + carbon intensity	Convex program + closed form	Prioritizes transparency and analytical solution

Result : Mathematical Model

Notation

For $t = 1, \dots, T$, let b_t denote inflexible demand (kWh), u_t flexible demand assigned to period t (kWh), s_t available PV generation (kWh), and c_t the grid-emission factor (kg CO₂e/kWh). Let U be total movable energy and $\bar{u}_t$ the maximum flexible demand permitted in period t .

Grid import is $g_t = \max\{0, b_t + u_t - s_t\}$.

The optimization problem is

$$\min_{\mathbf{u}} J(\mathbf{u}) = \sum_{t=1}^T c_t g_t + \lambda \sum_{t=1}^T g_t^2$$

subject to

$$\sum_{t=1}^T u_t = U$$

and

$$0 \leq u_t \leq \bar{u}_t, \quad t = 1, \dots, T.$$

where $\lambda \geq 0$ is a planning weight on uneven grid import, not a physical emission factor.

Availability restrictions may impose $u_t = 0$ outside permitted task windows.

Convexity

Proposition 1. If $c_t \geq 0$ and $\lambda \geq 0$, the optimization problem is convex.

Proof. The positive-part function $g_t(u_t) = \max\{0, b_t + u_t - s_t\}$ is convex. For $g \geq 0$, $f_t(g) = c_t g + \lambda g^2$ is convex and nondecreasing. Therefore $f_t(g_t(u_t))$ is convex by the composition rule. Summation preserves convexity, while the equality and box constraints define a convex feasible set. Hence every local minimizer is global. An equivalent quadratic-program representation introduces $g_t \geq 0$ and $g_t \geq b_t + u_t - s_t$.

Exact Two-Period Solution

Let x kWh be shifted from period 1 to period 2:

$$L_1(x) = B_1 - x, \quad L_2(x) = B_2 + x, \quad 0 \leq x \leq X.$$

Define $A = B_1 - s_1 > 0$ and $d = s_2 - B_2 \geq 0$. Then

$$g_1(x) = \max(0, A - x), \quad g_2(x) = \max(0, x - d).$$

When $d \leq x \leq A$, both imports are positive and

$$J(x) = c_1(A - x) + c_2(x - d) + \lambda[(A - x)^2 + (x - d)^2].$$

For $\lambda > 0$,

$$J'(x) = -c_1 + c_2 + 2\lambda(2x - A - d),$$

so the stationary point is

$$x_0 = \frac{A + d}{2} + \frac{c_1 - c_2}{4\lambda}.$$

A globally valid solution evaluates $x = 0$, $x = X$, feasible breakpoints $x = A$ and $x = d$, and x_0 whenever it lies in the corresponding active interval.

Illustrative Numerical Example

Assume $B_1 = 8$, $B_2 = 4$ kWh; $s_1 = 2$, $s_2 = 6$ kWh; $c_1 = 0.8$, $c_2 = 0.5$ kg CO₂e/kWh; $X = 6$ kWh; and $\lambda = 0.10$ kg CO₂e/(kWh²). These values are illustrative and were not measured at the author's institution.

Here $A = 6$, $d = 2$, and

$$x_0 = \frac{6 + 2}{2} + \frac{0.8 - 0.5}{4(0.10)} = 4.75 \text{ kWh.}$$

For $2 \leq x \leq 6$,

$$J(x) = 7.8 - 1.9x + 0.2x^2, \quad J''(x) = 0.4 > 0.$$

Thus $x^* = 4.75$ kWh uniquely minimizes the objective in this region.

Deterministic results for the hypothetical example.

Schedule	Shift x (kWh)	Grid imports (kWh)	Emissions (kg CO ₂ e)	J
Baseline	0	(6,0)	4.800	8.400
Solar saturation	2	(4,0)	3.200	4.800
Optimum	4.75	(1.25,2.75)	2.375	3.2875
Maximum shift	6	(0,4)	2.000	3.600

The modeled emissions reduction at the balanced optimum is

$$\frac{4.800 - 2.375}{4.800} \times 100 = 50.52\%.$$

This percentage is a property of the assumed numerical example and is not an estimate of actual savings at Phulsing Naik Mahavidyalaya.

Sensitivity Analysis

Within the two-import interval,

$$\frac{\partial x_0}{\partial (c_1 - c_2)} = \frac{1}{4\lambda}.$$

Thus, for fixed $\lambda > 0$, a larger carbon-intensity advantage for period 2 increases the optimal shift toward period 2. Increasing λ pulls the stationary point toward $(A + d)/2$, reflecting stronger preference for balanced grid imports. The shift limit X , PV surplus d , and deficit A can activate boundaries, so the unconstrained stationary point must always be checked against feasible intervals.

Practical Implementation and Reproducibility

A campus implementation should replace assumed inputs with synchronized hourly or sub-hourly meter data, measured PV output, defensible grid-emission factors, and task-specific operating windows. Flexible tasks should be indexed separately when they have different durations or precedence requirements. Storage can be added with state-of-charge dynamics; tariffs can be included as an additional objective term; and uncertain PV/load forecasts can be handled using scenario, robust, or stochastic extensions. Recent building-control literature demonstrates the importance of such flexibility and dynamic signals (Id Omar, Lê-Agopyan, and Sossan 2026; Li et al. 2026; Gou, Xu, and Sun 2025).

Conclusion

The numerical example is not an empirical case study. It excludes battery degradation, export remuneration, comfort constraints, rebound effects, forecast uncertainty, network losses, and embodied carbon. The quadratic term is a planning regularizer rather than measured CO₂ emissions. Moreover, average and marginal grid-emission factors can lead to different interpretations of load shifting. These limitations should be addressed before operational deployment.

A compact convex model can make the logic of solar-aware and carbon-aware demand shifting explicit. The formulation guarantees global optimality under stated assumptions and yields a closed-form two-period solution. The illustrative example shows how PV availability, temporal emission factors, and a grid-import balancing penalty jointly determine the optimum. The

principal value of the framework is methodological: it provides a transparent baseline for future empirical work using measured educational-building data.

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