VIDYABHARATI
Vidyabharati International Interdisciplinary Research Journal
ISSN 2319-4979

A MATHEMATICAL AND STATISTICAL ANALYSIS OF SEASONAL PM_{10} VARIABILITY IN NAGPUR CITY: A MULTI-LOCATION STUDY FOR ENVIRONMENTAL SUSTAINABILITY

Dr.V.P. Kadam
Assistant Professor And Head, Institution Name; Gopikabai Sitaram Gawande Mahavidyalaya,Umarkhed Maharashtra, India
kadam.v@gsgcollege.edu.in

Abstract
This study presents a mathematical and statistical assessment of seasonal variation in particulate matter with aerodynamic diameter below 10 micrometres (PM_{10}) across three types of monitoring locations in Nagpur City, India. The analysis uses a published eight-year dataset covering 2011–2018 and representing residential, industrial and commercial environments. The purpose is to demonstrate how simple statistical measures can convert environmental observations into interpretable evidence for sustainable air-quality management. Seasonal means, sample standard deviation, coefficient of variation, minimum and maximum values were calculated for each location. Annual PM_{10} means were then obtained by aggregating the four seasonal observations for each year, followed by a simple linear trend analysis. The results show that the industrial location had the highest overall annual mean PM_{10} concentration (117.06 µg/m³), followed by the residential location (101.63 µg/m³) and the commercial location (96.50 µg/m³). The post-monsoon season recorded the highest mean PM_{10} across the three locations (118.12 µg/m³), while the monsoon season had the lowest pooled seasonal mean (99.67 µg/m³). The industrial location also displayed the largest seasonal variability, particularly during the post-monsoon period, where the coefficient of variation was 20.94%. All calculated annual means were above the Indian annual PM_{10} National Ambient Air Quality Standard of 60 µg/m³. These findings indicate that seasonal and location-wise statistical analysis can provide a useful evidence base for environmental monitoring and urban planning. The study does not claim causality from the statistical patterns alone; instead, it identifies measurable patterns that can guide more detailed investigations involving meteorology, traffic, land use and emission sources.

***Keywords*:** *PM_{10}, air pollution, seasonal variation, mathematical analysis, descriptive statistics, coefficient of variation, Nagpur, environmental sustainability*

1. Introduction

Air pollution is a major environmental issue in rapidly developing urban areas. Among the different pollutants measured in ambient air, particulate matter is particularly important because its concentration can change with traffic, industrial activity, road dust, construction, weather conditions and seasonal atmospheric processes. PM_{10} refers to particulate matter with an aerodynamic diameter of 10 micrometres or less. Because these particles can remain suspended in the atmosphere and can enter the respiratory system, monitoring their concentration is an important part of urban environmental assessment.

Nagpur is a major urban centre in Maharashtra with residential, commercial, transport and industrial activities occurring within and around the city. Earlier monitoring-based research has documented variation in PM_{10}, SO_2 and NO_2 across residential, industrial and commercial locations in Nagpur over the period 2011–2018. Saini, Lataye and Motghare reported annual and seasonal variations and showed that the industrial monitoring location generally experienced higher PM_{10} concentrations than the residential and commercial locations. Their study also reported differences among seasons, making the dataset suitable for a focused mathematical re-analysis.

Mathematical and statistical methods are valuable in environmental research because large sets of observations can be reduced to a small number of meaningful indicators. The arithmetic mean provides a measure of central tendency, standard deviation indicates the spread of observations, and the coefficient of variation allows variability to be compared between datasets with different mean levels. Percentage differences and simple trend models can further support interpretation. Such methods do not replace environmental measurements; rather, they provide a systematic way to understand the measurements.

The present study therefore focuses specifically on the seasonal behaviour of PM_{10} in Nagpur. Instead of attempting to establish a causal relationship between PM_{10} and individual emission sources, the paper examines the numerical structure of the published data. This approach is useful for an interdisciplinary conference because it connects mathematics, statistics and environmental science through a transparent analytical framework.

2. Objectives of the Study

- To analyse the seasonal PM_{10} concentration at residential, industrial and commercial monitoring locations in Nagpur.
- To calculate mean, standard deviation and coefficient of variation for each location and season.
- To obtain annual mean PM_{10} values from the four seasonal observations for 2011–2018.
- To examine the direction and strength of simple linear annual trends.
- To compare annual mean concentrations with the Indian annual PM_{10} NAAQS value of 60 $\mu g/m^3$.
- To discuss how mathematical evidence can support environmental monitoring and sustainable urban planning.

3. Literature Review

Saini, Lataye and Motghare (2023) examined PM_{10}, SO_2 and NO_2 in and around Nagpur during 2011–2018 using residential, industrial and commercial monitoring locations. Their results demonstrated substantial variation in PM_{10} and reported annual average PM_{10} values of 101.87, 115.37 and 98.75 $\mu g/m^3$ for the residential, industrial and commercial locations, respectively, over the study period. The same work also examined seasonal behaviour and atmospheric indicators such as planetary boundary layer height and ventilation coefficient.

CPCB monitoring reports have consistently treated PM_{10} as one of the principal pollutants in the National Ambient Air Quality Monitoring Programme. CPCB reports use an annual PM_{10} standard of 60 $\mu g/m^3$ for residential, industrial, rural and other areas under the National Ambient Air Quality Standards. The standards provide a regulatory reference for evaluating annual ambient concentrations.

The World Health Organization's 2021 global air quality guidelines provide a health-based annual PM_{10} guideline value of 15 $\mu g/m^3$. This value is not identical to India's regulatory NAAQS; the two should therefore not be treated as interchangeable. The present study uses the Indian NAAQS for regulatory comparison and mentions the WHO guideline only as a broader health-oriented reference.

Statistical analysis of atmospheric observations commonly uses descriptive measures, graphical exploration, trend analysis and uncertainty assessment. Wilks (2011) describes the role of statistical methods in summarizing, analysing and interpreting atmospheric data, while standard regression texts provide the mathematical basis for simple linear trend estimation. These principles motivate the statistical framework used in this study.

4. Research Work

4.1 Study Area and Data Source

The study is based on Nagpur City, Maharashtra, India. Three monitoring environments are considered: Station I representing a residential location, Station II representing an industrial location, and Station III representing a commercial location. The analysis uses the seasonal PM_{10} observations reported for 2011–2018 by Saini et al. (2023). The dataset contains four seasons—winter, summer, monsoon and post-monsoon—for each of the three locations and eight years.

4.2 Dataset

Location	Season	Years	Observations	Minimum	Maximum
Residential (Station I)	Winter	2011–2018	8	89	103
Residential (Station I)	Summer	2011–2018	8	82	118
Residential (Station I)	Monsoon	2011–2018	8	78	120
Residential (Station I)	Post-Monsoon	2011–2018	8	84	129
Industrial (Station II)	Winter	2011–2018	8	102	141
Industrial (Station II)	Summer	2011–2018	8	87	135
Industrial (Station II)	Monsoon	2011–2018	8	84	140
Industrial (Station II)	Post-Monsoon	2011–2018	8	95	182
Commercial (Station III)	Winter	2011–2018	8	84	109
Commercial (Station III)	Summer	2011–2018	8	77	118
Commercial (Station III)	Monsoon	2011–2018	8	74	124
Commercial (Station III)	Post-Monsoon	2011–2018	8	92	121

4.3 Mathematical and Statistical Method

The following measures were applied:

- Arithmetic Mean: $\bar{x} = (\Sigma x_i)/n$, where x_i is a PM_{10} observation and n is the number of observations.
- Sample Standard Deviation: $s = \sqrt{[\Sigma(x_i - \bar{x})^2/(n-1)]}$.
- Coefficient of Variation: $CV = (s/\bar{x}) \times 100$. A higher CV indicates greater relative variability.
- Annual Mean: for each year and location, the mean of the four seasonal observations was calculated.
- Simple Linear Trend: $y = a + bx$, where b represents the estimated annual change in the calculated annual mean series.
- Coefficient of Determination (R^2): used to describe how closely the annual observations follow the fitted linear relationship. With only eight annual observations, this is treated as descriptive evidence rather than a forecasting model.



Figure 1. Step-by-step methodology used in the study.

4.4 Descriptive Statistical Results

Location	Season	Mean	SD	CV (%)	Min	Max
Residential (Station I)	Winter	98.00	4.50	4.60	89	103
Residential (Station I)	Summer	95.88	13.75	14.34	82	118
Residential (Station I)	Monsoon	96.50	14.92	15.46	78	120
Residential (Station I)	Post-Monsoon	116.12	14.67	12.63	84	129
Industrial (Station II)	Winter	118.50	14.42	12.17	102	141
Industrial (Station II)	Summer	108.12	17.46	16.15	87	135
Industrial (Station II)	Monsoon	111.50	18.69	16.77	84	140
Industrial (Station II)	Post-Monsoon	130.12	27.25	20.94	95	182
Commercial (Station III)	Winter	94.50	8.26	8.74	84	109
Commercial (Station III)	Summer	92.38	14.32	15.50	77	118
Commercial (Station III)	Monsoon	91.00	16.51	18.14	74	124
Commercial (Station III)	Post-Monsoon	108.12	9.23	8.54	92	121

The descriptive results show a clear spatial pattern. The industrial location has the highest mean PM_{10} in every season considered: 118.50 µg/m³ in winter, 108.12 µg/m³ in summer, 111.50 µg/m³ during monsoon and 130.12 µg/m³ in post-monsoon. The residential location has corresponding seasonal means of 98.00, 95.88, 96.50 and 116.12 µg/m³. The commercial location records 94.50, 92.38, 91.00 and 108.12 µg/m³. Thus, the post-monsoon period has the highest mean concentration at all three locations.

The coefficient of variation provides additional information. The industrial post-monsoon series has a CV of 20.94%, the largest among the location-season combinations. This indicates that the industrial post-monsoon observations fluctuate considerably around their mean. The residential winter series has a CV of only 4.60%, showing comparatively stable values across the eight years. Such differences would not be visible from mean values alone.

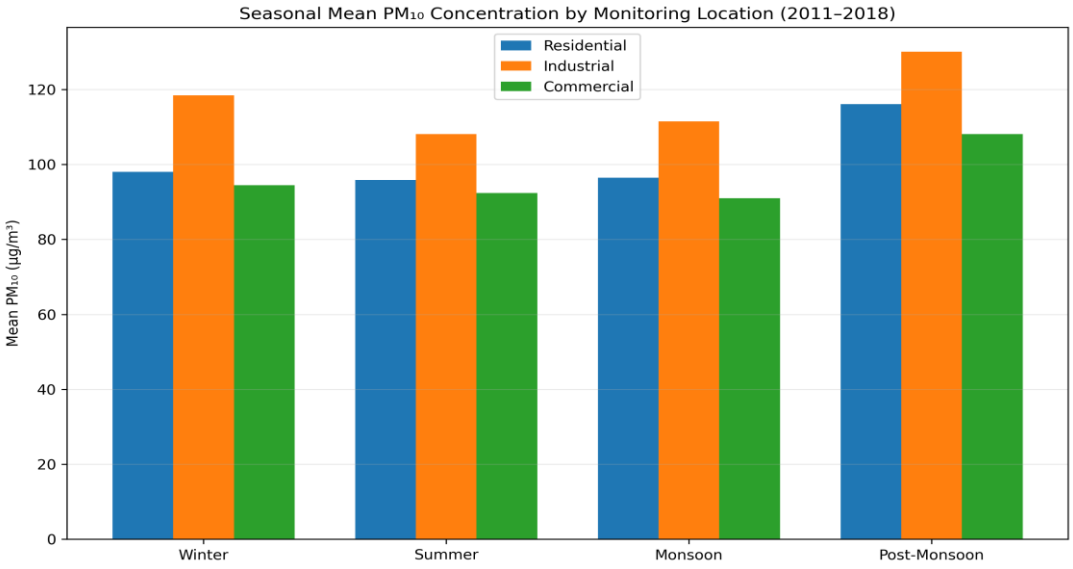


Figure 2. Seasonal mean PM_{10} concentration by monitoring location, 2011–2018.

4.5 Annual Mean and Trend Analysis

Location	Mean Annual	SD	CV (%)	Min	Max	Slope/year	R^2
Residential (Station I)	101.62	7.48	7.36	88.00	110.25	-0.02	0.000
Industrial (Station II)	117.06	12.72	10.87	98.50	138.50	-2.70	0.270
Commercial (Station III)	96.50	6.73	6.98	88.50	107.75	1.63	0.352

The industrial location has the highest overall annual mean of 117.06 µg/m³. Its calculated annual series ranges from 98.50 to 138.50 µg/m³. The residential location has an overall annual mean of 101.63 µg/m³, while the commercial location has 96.50 µg/m³. These values are consistent with the general spatial pattern reported in the underlying published study, while the exact values here are calculated by averaging the four seasonal observations supplied in its seasonal table.

The fitted linear slopes are small relative to the year-to-year fluctuations. The residential series has a slope of approximately −0.02 µg/m³ per year and R^2 close to zero, indicating no useful linear direction in the eight calculated annual observations. The industrial series has a slope of −2.70 µg/m³ per year with R^2 = 0.270, while the commercial series has a slope of +1.63 µg/m³ per year with R^2 = 0.352. These results should not be interpreted as proof of long-term improvement or deterioration. They only describe the linear component of a short eight-year series.

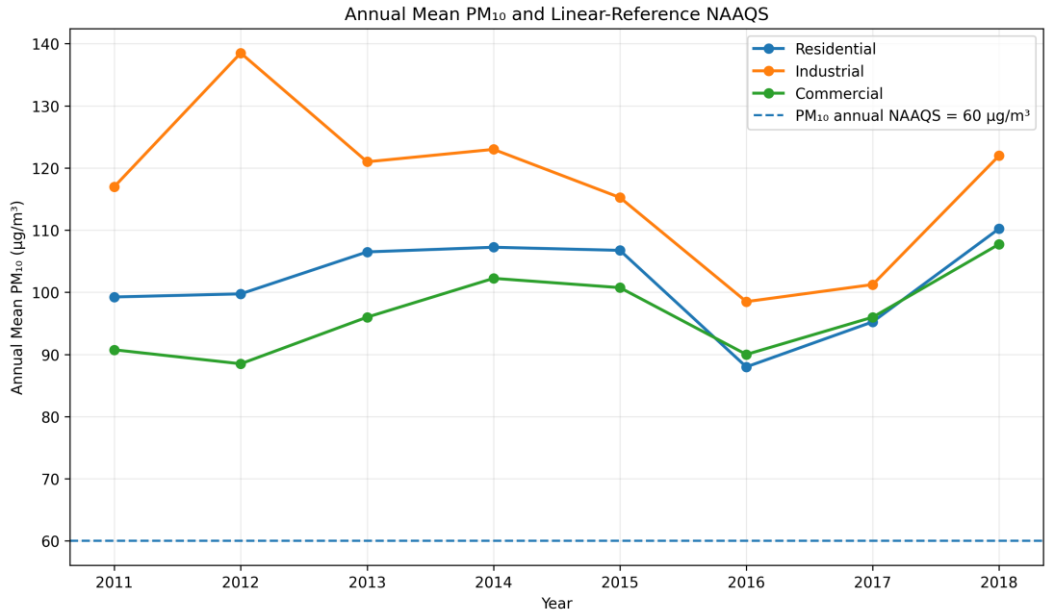


Figure 3. Calculated annual mean PM_{10} series and the Indian annual NAAQS reference of 60 µg/m³.

4.6 Comparison with the Indian NAAQS

Location	Overall annual mean (µg/m³)	Indian annual NAAQS (µg/m³)	Mean / NAAQS
Residential (Station I)	101.62	60	1.69×
Industrial (Station II)	117.06	60	1.95×
Commercial (Station III)	96.50	60	1.61×

All three locations have calculated overall annual means above the Indian annual PM_{10} NAAQS of 60 µg/m³. The industrial location is approximately 1.95 times the standard on the basis of its eight-year mean, compared with approximately 1.69 times for the residential location and 1.61

times for the commercial location. This comparison is an aggregate descriptive indicator and should not be interpreted as a health-risk calculation.

5. Discussion

The strongest feature of the dataset is the consistent elevation of PM_{10} at the industrial location. The mean concentration at this location is higher than the other two locations in all four seasons. The result is compatible with the broader monitoring literature for Nagpur, but the statistical analysis alone cannot identify the precise contribution of industrial emissions, road dust, traffic or meteorological conditions.

The post-monsoon season has the highest mean PM_{10} at all locations. In the industrial location, the post-monsoon mean is 130.12 µg/m³, compared with 108.12 µg/m³ in summer and 111.50 µg/m³ during monsoon. The underlying published study also discusses atmospheric conditions and seasonal variation. The present analysis therefore treats seasonality as an observed pattern rather than assigning a single cause.

The lower monsoon average across the three locations is consistent with a reduction in the persistence of suspended particulate matter under wetter atmospheric conditions, but the dataset alone does not permit a formal causal test of rainfall or atmospheric dispersion. A stronger future study should combine PM_{10} measurements with rainfall, wind speed, temperature, humidity, boundary-layer height and traffic indicators.

From a mathematical perspective, the study demonstrates why multiple measures are necessary. Mean concentration identifies the level, standard deviation describes absolute variation, CV describes relative variation, and the linear slope provides a simple measure of direction. The R^2 statistic adds information about how well a straight-line model represents the observations. Together, these measures provide a compact but informative environmental profile.

6. Environmental and Sustainability Implications

Regular seasonal analysis can help environmental agencies identify periods and locations requiring closer monitoring. The industrial location and the post-monsoon season emerge as important parts of the dataset for further investigation.

Mathematical summaries can support transparent communication between environmental scientists, urban planners and local decision makers. Instead of relying only on individual high or low readings, decision makers can examine mean levels, variability and repeated seasonal patterns.

For sustainable urban management, the statistical findings can be combined with source-apportionment studies, traffic data, dust-control programmes, industrial emission inventories and meteorological observations. Such integration would provide stronger evidence for targeted interventions than statistical analysis alone.

The comparison with the Indian NAAQS also shows the importance of distinguishing regulatory standards from health-based international guidelines. The WHO 2021 annual PM_{10} guideline is 15 µg/m³, whereas India's annual NAAQS for the relevant area category is 60 µg/m³. These values serve different policy and scientific purposes and should be reported separately.

7. Limitations

- The analysis is based on secondary published data rather than newly collected measurements.
- The seasonal dataset contains only eight years and therefore provides a relatively small number of annual observations for trend analysis.
- Only PM_{10} is analysed in the present paper; SO_2, NO_2 and meteorological variables are not included in the statistical model.
- Annual NAAQS comparison is appropriate for annual mean concentrations; the 60 µg/m³ annual standard is not used as a direct regulatory threshold for individual seasonal means.
- The analysis identifies associations and patterns but does not establish causal relationships between PM_{10} and specific emission sources.

8. Future Scope

Future work can extend this study by incorporating $PM_{2.5}$, NO_2, SO_2, CO and meteorological variables; applying Mann–Kendall and Sen's slope methods for monotonic trend assessment; using time-series models for longer datasets; developing GIS-based spatial maps; and integrating traffic and land-use information. A larger dataset could also support multivariate regression or machine-learning models for PM_{10} prediction. Such extensions would move the analysis from descriptive statistics toward environmental forecasting and decision support.

9. Conclusion

This study demonstrates a simple and reproducible mathematical framework for analysing seasonal PM_{10} variability in Nagpur City. Using an eight-year published dataset from three monitoring environments, the analysis found the highest overall annual mean at the industrial location, followed by the residential and commercial locations. The post-monsoon season showed the highest mean PM_{10} concentration across all three locations, while the industrial post-monsoon series showed the largest relative variability. All calculated annual means were above India's annual PM_{10} NAAQS reference of 60 $\mu g/m^3$. The linear trend results were mixed and relatively weak, showing why a short time series should not be converted into a strong forecast or causal claim. Overall, the study illustrates how elementary statistical tools can transform environmental observations into useful evidence for monitoring, comparison and sustainability-oriented research.

References

1.Saini, D., Lataye, D., & Motghare, V. M. (2023). Studies on the variation in concentrations of respirable suspended particulate matter (PM_{10}), NO_2 and SO_2 in and around Nagpur. Mausam, 74(3), 761–786. https://doi.org/10.54302/mausam.v74i3.828

2.Central Pollution Control Board. (2016). Annual report 2015–16. Ministry of Environment, Forest and Climate Change, Government of India.

3.Central Pollution Control Board. (2018). Annual report 2017–18. Ministry of Environment, Forest and Climate Change, Government of India.

4.Central Pollution Control Board. (2019). National ambient air quality monitoring programme: Air quality status and trends. Government of India.

5.Central Pollution Control Board. (2020). Annual report 2019–20. Ministry of Environment, Forest and Climate Change, Government of India.

6.Maharashtra Pollution Control Board. (2019). Air quality status of Maharashtra 2018–19. Government of Maharashtra.

7.Maharashtra Pollution Control Board. (2024). Environmental impact and social assessment report: Nagpur. Government of Maharashtra.

8.World Health Organization. (2021). WHO global air quality guidelines: Particulate matter ($PM_{2.5}$ and PM_{10}), ozone, nitrogen dioxide, sulfur dioxide and carbon monoxide. World Health Organization.

9.World Health Organization. (2022). WHO releases new repository of resources for air quality management. World Health Organization.

10.Wilks, D. S. (2011). Statistical methods in the atmospheric sciences (3rd ed.). Academic Press.

11.Montgomery, D. C., Peck, E. A., & Vining, G. G. (2021). Introduction to linear regression analysis (6th ed.). Wiley.

12.Box, G. E. P., Jenkins, G. M., Reinsel, G. C., & Ljung, G. M. (2015). Time series analysis: Forecasting and control (5th ed.). Wiley.

13.Rao, C. R. (2009). Linear statistical inference and its applications (2nd ed.). Wiley.

Seinfeld, J. H., & Pandis, S. N. (2016). Atmospheric chemistry and physics: From air pollution to climate change (3rd ed.). Wiley.

14.United Nations. (2015). Transforming our world: The 2030 agenda for sustainable development. United Nations.