# ECONOMIC PRODUCTION QUANTITY MODEL FOR INTEGRATED INVENTORY CONTROL

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**Abstract**

*Here attempt has been made to develop EPQ model for items which deteriorates with respect to time. The demand pattern used is mix type. Here inventory dependent demand rate is used when production is in process and assumes demand to be constant after maximum inventory level reaches. The optimum solution of the model is derived by using simple differential calculus method. The effect of rate at which inventory get consumed is discussed in this model. In this model the cost function shows the convexity. Mathematical Model gets verified by using numerical example and Sensitivity analysis.*

*Keywords: EPQ, Inventory dependent.*

## 1. Introduction

The EPQ models are extensively used to control inventories due to its ease in application. Traditional models assume that only perfect quality products produced during production and demand rate is also constant. But actually, it is not possible. When product is in maturity stage then constant demand may possible. Buying capacity of the customer increases if the stock is in bulk amount. In perishable items deterioration is the common phenomenon. Numerous researches had been carried out by number of researchers by considering different rate of demand and deterioration. Chung [1] developed model for production process which are imperfect and considered different rates of deterioration. Cheng *et.al* [2] suggested the EOQ model with demand varying with respect to stock available for deteriorating items. Their investigation shows that optimal solution is highly sensitive to demand and selling price. Chiu [3] presented EPQ model under random machine breakdown and consider corrective maintenance period as fixed. Gede and Hui [4] assumes that the demand parameter fluctuating with time and considered purchase cost as positive. Gerg *et.al.* [5] used LIFO policy to investigate the effect of imperfect production process. Jain and Sharma [6] studied inventory model for price and inventory dependent demand. Widyadana and Krishnamoorthy [7] considered machine failure with random repair time. Khedlekar [8] examined the effect of machine breakdown which occurs randomly on optimal run time and average cost with constant demand and backorder. Lin ad Gong [9] considers the effect of reworking on the EPQ model with backlogging. Lin ad Gong [10] taken random unavailability of machines and shortage due to it. Mishra and Singh [11] developed inventory model with exponential varying demand which depends on stock available and items deteriorates cubically. Min *et.al*. [12] suggested the inventory dependent demand which is price sensitive to construct the inventory model with two credit policies and price negotiation. Rosenblatt [13] considered the regular rework of defective items to convert them into finished goods and presented the defect sales return. Teng *et.al*. [14] presented an EPQ model for ramp type production and demand rate and considering deterioration as non-instantaneous and Weibull in nature. Shah and Patel [15] considered disrupted production system to establish model for an exponential demand. Teng *et.al* [16] assumes demand rate to be dependent on the retailer's current inventory level and developed a replenishment model with trade credit for deteriorating items. From the literature review revels various demand pattern discussed by number of researchers for different production systems. Here in this article we assumed inventory dependent demand when production is in process and constant demand when sufficient stock gets build up. This paper is in four sections. Literature is in section 1. Mathematical model development is discussed in section 2. Section 3 has numerical analysis and finally concluded in section 4 .

## 2. Model development

Assumptions: -

Model is developed as follows with some basic assumptions.

a) Rate of production to be constant.
b) Demand rate is less than production rate.
c) Constant rate and cost of deterioration.
d) Constant Inventory keeping cost
e) Items will deteriorate as they enter in stock.
f) No shortages

Notation: -
$I_1$ – Items in Stock in build-up period.
$I_2$ – Items in Stock when production stops.
$T_1$– Inventory build-up time.
$T_2$ – Time where production stops.
S –Production rate.
C – Basic demand rate.
σ – Deterioration rate.
h – Holding cost.
$C_d$ – Deterioration cost.
T – Total Production time.
TC – Total cost of Production
TCT – per unit time cost of items.

The production starts initially. As production is in process so inventory will be developed from time t = 0 to t = $T_1$. During inventory build-up period demands depends on inventory. Maximum stock will be at time t=$T_1$. After time $T_1$ production stops and available inventory will be sufficient to fulfil the demand. The deterioration starts when item produced became the part of inventory.

$$\frac{dI_1(t)}{dt} = P - D - \alpha I_1(t) - \theta I_1(t) \qquad 0 \le t \le T_1 \qquad (1)$$
$$\frac{dI_2(t)}{dt} = -D - \theta I_2(t) \qquad 0 \le t \le T_2 \qquad (2)$$
$$\frac{dI_3(t)}{dt} = P - D - \alpha I_3(t) - \theta I_3(t) \qquad T_2 \le t \le T_3 \qquad (3)$$
$$\frac{dI_4(t)}{dt} = -D - \theta I_4(t) \qquad 0 \le t \le T_4 \qquad (4)$$

*Initial boundary conditions associated with this equations are,*
*when* $t = 0$ *then* $I_1(t) = 0$ ; *when* $t = T_2$ *then* $I_2(t) = I_2(T_2)$;
*when* $t = T_3$ *then* $I_3(t) = I_3(T_3)$ ; *when* $t = T_4$ *then* $I_4(t) = 0$

$$I_1(t) = \frac{P-D}{\alpha+\theta}\left[1 - e^{-(\alpha+\theta)t}\right] \qquad 0 \le t \le T_1 \qquad (5)$$
$$I_2(t) = \frac{-D}{\theta} + \left[I_2(T_2) + \frac{D}{\theta}\right] e^{\theta(T_2 - t)} \qquad 0 \le t \le T_2 \qquad (6)$$
$$I_3(t) = \frac{P-D}{\alpha+\theta} + \left[I_3(T_2) - \frac{P-D}{\alpha+\theta}\right] e^{(\alpha+\theta)(T_2 - t)} \qquad T_2 \le t \le T_3 \qquad (7)$$
$$I_4(t) = \frac{D}{\theta}\left[e^{\theta(T_4 - t)} - 1\right] \qquad 0 \le t \le T_4 \qquad (8)$$

*From equation* (5) & (6), *by using boundary conditions* $I_1(T_1) = I_2(0)$

$$\frac{P-D}{\alpha+\theta}\left[1 - e^{-(\alpha+\theta)T_1}\right] = \frac{-D}{\theta} + \left[I_2(T_2) + \frac{D}{\theta}\right] e^{\theta(T_2)}$$
$$T_2 \approx \frac{P-D}{I_2(T_2)\theta + D} T_1 - \left(\frac{\alpha+\theta}{2\theta}\right)\frac{1}{I_2(T_2)\theta + D} T_1^{\,2} + \frac{D}{\theta[I_2(T_2)\theta + D]} - \frac{1}{\theta}$$
$$T_2 \approx K_1 T_1 - K_2 T_1^{\,2} + K_3 \qquad (9)$$

*From equation* (7) & (8), *by using boundary conditions* $I_3(T_3) = I_4(0)$

$$\frac{P-D}{\alpha+\theta} + \left[I_3(T_2) - \frac{P-D}{\alpha+\theta}\right] e^{(\alpha+\theta)(T_2 - T_3)} = \frac{D}{\theta}\left[e^{\theta(T_4)} - 1\right]$$
$$T_4 = \frac{1}{D}\left\{\frac{(P-D) + (\alpha+\theta)\left[I_3(T_2) - \frac{P-D}{\alpha+\theta}\right]}{(\alpha+\theta)}\right\}\left[1 + (\alpha+\theta)(T_2 - T_3)\right]$$
$$T_4 = K_4\left[1 + (\alpha+\theta)\left(K_1 T_1 - K_2 T_1^{\,2} + K_3 - T_3\right)\right] \qquad (10)$$

Total inventory is given by sum of inventory in up time and inventory in down time

$$TI = \int_0^{T_1} I_1(t)dt + \int_0^{T_2} I_2(t)\,dt + \int_{T_2}^{T_3} I_3(t)\,dt + \int_0^{T_4} I_4(t)dt \qquad (11)$$

By using Taylors series approximations for exponential function and neglecting terms of higher power of $\alpha$ *and* $\theta$, *the approximate total inventary can be express as,*

$$TI = \left(\frac{P-D}{2}\right){T_1}^2 + T_2 I_2(T_2) + [I_2(T_2)\theta + D]\frac{{T_2}^2}{2} + I_3(T_2)(T_3 - T_2) + [(P-D) - I_3(T_2)(\propto + \theta)]\frac{(T_3 - T_2)^2}{2} + \frac{D}{2}{T_4}^2 \quad (12)$$

Total cost = Set up cost + Holding cost

$TC = A + H(TI)$

From Equation (9) and (10) equation (12) implies

$$TC = A + H\Big\{\left(\frac{P-D}{2}\right){T_1}^2 + \left(K_1T_1 - K_2{T_1}^2 + K_3\right)I_2(T_2) + K_5\left(K_1T_1 - K_2{T_1}^2 + K_3\right)^2 + I_3(T_2)\left(T_3 - K_1T_1 + K_2{T_1}^2 - K_3\right) + K_6\left[{T_3}^2 - 2K_1K_3T_1 + \left({K_1}^2 + 2K_2K_3\right){T_1}^2 - 2K_1K_2{T_1}^3 + {K_3}^2 - 2\left(K_1T_1T_3 - K_2{T_1}^2T_3 + K_3T_3\right) + {K_2}^2{T_1}^4\right] + \frac{D}{2}{K_4}^2\left[1 + (\propto + \theta)\left(K_1T_1 - K_2{T_1}^2 + K_3 - T_3\right)\right]\Big\}$$

$$TC = A + M_1T_1 + M_2{T_1}^2 - M_3{T_1}^3 + M_4T_3 + M_5{T_3}^2 + M_6T_1T_3 - M_7{T_1}^2T_3 + M_8 + M_9{T_1}^4 \quad (13)$$

$\text{Total cost per unit time} = \text{TCT} = \frac{TC}{T}$

$\text{Production cycle time} = \text{T} = T_1 + T_2 + T_3 + T_4$

$$T = [1 + K_1 + K_1K_4(\propto + \theta)]T_1 + [-K_2 - (\propto + \theta)K_2K_4]{T_1}^2 + [1 - K_4(\propto + \theta)]T_3 + [K_3 + K_4 + K_3K_4(\propto + \theta)]$$

$$T = M_{10}T_1 + M_{11}{T_1}^2 + M_{12}T_3 + M_{13}$$

$$TCT = \frac{A + M_1T_1 + M_2{T_1}^2 - M_3{T_1}^3 + M_4T_3 + M_5{T_3}^2 + M_6T_1T_3 - M_7{T_1}^2T_3 + M_8 + M_9{T_1}^4}{M_{10}T_1 + M_{11}{T_1}^2 + M_{12}T_3 + M_{13}} \quad (14)$$

The optimum production up time can be derived by satisfying the equation

$$\frac{\partial TCT}{\partial T_1} = 0 \ \ and \ \ \frac{\partial TCT}{\partial T_3} = 0 \quad (15)$$

Ignoring the higher power terms of $T_1$ and $T_3$

$$(2M_2M_{13} - 2M_{11}A - 2M_8M_{11})T_1 + (M_2M_{10} - 3M_3M_{13} - M_1M_{13}){T_1}^2 + (2M_2M_{12} - 2M_7M_{13} - 2M_4M_{11})T_1T_3 + (M_1M_{12} + M_6M_{13} - M_4M_{10})T_3 + (M_6M_{12} - M_5M_{10}){T_3}^2 + (M_1M_{13} - AM_{10} + M_8) = 0$$

$$(M_6M_{10} + M_6M_{13} + M_1M_{12})T_1 + (M_4M_{11} - M_7M_{10} - M_7M_{13} + M_7){T_1}^2 + (2M_6M_{12})T_1T_3 + (2M_5M_{10} + 2M_4M_{12} + 2M_5M_{13})T_3 + (3M_5M_{12}){T_3}^2 + (M_4M_{10} + M_4M_{13} + M_8M_{12}) = 0$$

$$a_1T_1 + b_1{T_1}^2 + c_1T_1T_3 + d_1T_3 + e_1{T_3}^2 + f_1 = 0 \quad (16)$$

$$a_2T_1 + b_2{T_1}^2 + c_2T_1T_3 + d_2T_3 + e_2{T_3}^2 + f_2 = 0 \quad (17)$$

Solve equation (16) for $T_3$, if $e_1 = 0$ the equation (16) is linear in $T_3$

$$T_3 = -\frac{a_1T_1 + b_1{T_1}^2 + f_1}{c_1T_1 + d_1} \quad provided\ c_1T_1 + d_1 \neq 0 \quad (18)$$

*Put equation (18) in to (17) we get the value of* $T_1$

$$\left(2a_2c_1d_1 + b_2{d_1}^2 + c_1d_2a_1 + d_1c_2a_1 + d_1d_2b_1 + c_1c_2f_1 + {a_1}^2 + {b_1}^2 + 2f_1b_1 + {c_1}^2f_2\right){T_1}^2 + \left(a_2{d_1}^2 + c_1d_2f_1 + d_1c_2f_1 + d_1d_2a_1 + 2a_1f_1c_2 + 2c_1f_1d_1\right)T_1 + \left(d_1d_2f_1 + e_2{f_1}^2 + {d_1}^2f_2\right) = 0$$

$$p{T_1}^2 + qT_1 + r = 0 \quad (19)$$

The Approximate solution can be obtained by considering positive roots of the equation (19)

Where,

$$K_1 = \frac{P-D}{I_2(T_2)\theta + D}$$

$$K_2 = \left(\frac{\propto + \theta}{2\theta}\right)\frac{1}{I_2(T_2)\theta + D}$$

$$K_3=\frac{D}{\theta[I_2(T_2)\theta+D]}-\frac{1}{\theta}$$

$$K_4=\frac{1}{D}\left\{\frac{(P-D)+(\propto+\theta)\left[I_3(T_2)-\frac{P-D}{\propto+\theta}\right]}{(\propto+\theta)}\right\}$$

$$K_5=\frac{[I_2(T_2)\theta+D]}{2}$$

$$K_6=\frac{[(P-D)-I_3(T_2)(\propto+\theta)]}{2}$$

$$M_1=H\{K_1I_2(T_2)-2(K_5+K_6)K_1K_3-K_1I_3(T_2)+DK_4{}^2[(\propto+\theta)K_1+(\propto+\theta)^2(K_1K_3-K_2K_3)]\}$$

$$M_2=H\left\{\frac{P-D}{2}-K_2I_2(T_2)+(K_5+K_6)(K_1{}^2+K_2K_3)-K_2I_3(T_2)+\frac{DK_4{}^2}{2}[2(\propto+\theta)K_2+(\propto+\theta)^2K_1{}^2]\right\}$$

$$M_3=HK_1K_2(2K_5+2K_6+DK_4{}^2(\propto+\theta)^2)$$

$$M_4=H\{I_3(T_2)-2K_3K_6-DK_4{}^2[(\propto+\theta)+(\propto+\theta)^2K_3]\}$$

$$M_5=H\left[K_6+\frac{1}{2}DK_4{}^2(\propto+\theta)^2\right]$$

$$M_6=H\left[-2K_1K_6+\frac{1}{2}DK_4{}^2(\propto+\theta)^2(K_2-K_1)\right]$$

$$M_7=HK_2K_6$$

$$M_8=H\left\{K_3I_2(T_2)-K_3I_3(T_2)+K_3{}^2\left[K_5+K_6+\frac{1}{2}DK_4{}^2(\propto+\theta)^2\right]+\frac{1}{2}DK_4{}^2\right\}$$

$$M_9=HK_2{}^2\left[K_5+K_6+\frac{1}{2}DK_4{}^2(\propto+\theta)^2\right]$$

$$M_{10}=[1+K_1+K_1K_4(\propto+\theta)]$$

$$M_{11}=[-K_2-(\propto+\theta)K_2K_4]$$

$$M_{12}=[1-K_4(\propto+\theta)]$$

$$M_{13}=[K_3+K_4+K_3K_4(\propto+\theta)]$$

$$a_1=2M_2M_{13}-2M_{11}A-2M_8M_{11}$$

$$b_1=M_2M_{10}-3M_3M_{13}-M_1M_{13}$$

$$c_1=2M_2M_{12}-2M_7M_{13}-2M_4M_{11}$$

$$d_1=M_1M_{12}+M_6M_{13}-M_4M_{10}$$

$$e_1=M_6M_{12}-M_5M_{10}$$

$$f_1=M_1M_{13}-AM_{10}+M_8$$

$$a_2=M_6M_{10}+M_6M_{13}+M_1M_{12}$$

$$b_2=M_4M_{11}-M_7M_{10}-M_7M_{13}+M_7$$

$$c_2=2M_6M_{12}$$

$$d_2=2M_5M_{10}+2M_4M_{12}+2M_5M_{13}$$

$$e_2=3M_5M_{12}$$

$$f_2=M_4M_{10}+M_4M_{13}+M_8M_{12}$$

$$p=2a_2c_1d_1+b_2d_1{}^2+c_1d_2a_1+d_1c_2a_1+d_1d_2b_1+c_1c_2f_1+a_1{}^2+b_1{}^2+2f_1b_1+c_1{}^2f_2$$

$$q=a_2d_1{}^2+c_1d_2f_1+d_1c_2f_1+d_1d_2a_1+2a_1f_1c_2+2c_1f_1d_1$$

$$r=d_1d_2f_1+e_2f_1{}^2+d_1{}^2f_2$$

## 3. Model Validation

Developed model has been validated by using numerical and sensitivity analysis. For validation data is adopted from Teng *et al*. (16).

A is set up cost and considered as Rs.30 per production cycle, $C_d$ = Rs.4 /unit /unit time, , h = Rs.2.5 / unit, unit time, S = 2600 units / unit time, C = 1400 units / unit time, β = 0.5, σ = 0.1

Fig. 1 shows the relation between total cost per unit time and production time, which is convex in nature. The best possible value of production time is 0.124. The most desirable value of TCT is Rs.231.83. Parameters are changed one by one by -40% to +40%.

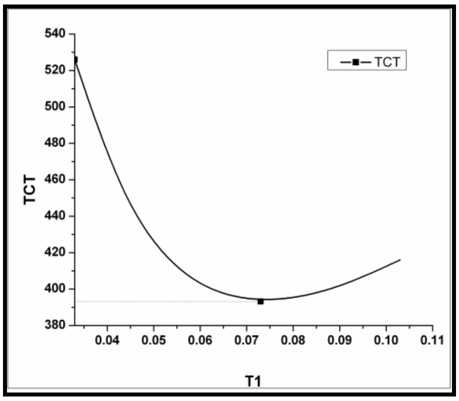


Fig.1: $T_1$ v/s TCT

| Parameters | Parameter changes | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | -40% | | -20% | | 20% | | 40% | |
| | T1 | TCT | T1 | TCT | T1 | TCT | T1 | TCT |
| P | 0.398 | 120.644 | 0.187 | 192.241 | 0.093 | 257.899 | 0.074 | 276.17 |
| D | 0.075 | 247.55 | 0.098 | 235.61 | 0.158 | 219.18 | 0.204 | 197.3 |
| α | 0.108 | 267.186 | 0.115 | 249.748 | 0.136 | 211.786 | 0.152 | 189.64 |
| H | 0.19 | 151.847 | 0.147 | 196.5 | 0.109 | 263.818 | 0.104 | 278.026 |

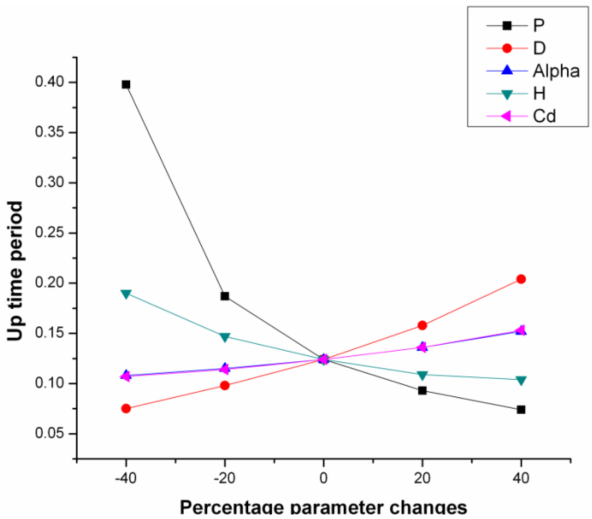


Fig.2 shows the relation between demand parameter and holding cost. But at the same time total cost goes on decreasing with increase in demand parameter. Relation shows that demand is higher at higher inventory level. Figure shows relation between production time and demand parameter. Production time increases due to increase in demand parameter. It means increase in inventory dependent consumption rate parameter increase demand. From fig 2 it is clear that demand parameter plays important role in inventory management.

## 4. Conclusion

EPQ model has been developed and studied theoretically for deteriorating items. By sensitivity analysis has given important observations. Stock dependent demand is used during stock build up period and constant demand after maximum inventory. Total cost per unit time decreases aa demand parameter increases. At the same time inventory holding cost increases due to maximum inventory. So proper selection of this parameter is important in decision making.

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